פתרון בוחן

פתרון שאלה 1: (א) נחשב תחילה $[I]_B^E$

$$[I]_B^E = \left([I]_E^B\right)^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ -1 & 1 & 3 \\ 3 & -1 & -2 \end{pmatrix}^{-1} = \dots = \begin{pmatrix} 1 & 0 & 0 \\ 7 & -2 & -3 \\ -2 & 1 & 1 \end{pmatrix}$$

כעת,

$$(2,1,-1)\begin{pmatrix} 1 & 0 & 0 \\ 7 & -2 & -3 \\ -2 & 1 & 1 \end{pmatrix} = (11,-3,-4)$$

ולכן

$$\boxed{f(x,y,z) = 11x - 3y - 4z}$$

(ב) נחשב בס"ד את הבסיס הדואלי $\{\varphi_1, \varphi_2, \varphi_3\}$ של B.

φ_1: $(1,0,0)[I]_B^E = (1,0,0)\begin{pmatrix} 1 & 0 & 0 \\ 7 & -2 & -3 \\ -2 & 1 & 1 \end{pmatrix} = (1,0,0)$

$$\Rightarrow \boxed{\varphi_1(x,y,z) = x}$$

φ_2: $(0,1,0)[I]_B^E = (0,1,0)\begin{pmatrix} 1 & 0 & 0 \\ 7 & -2 & -3 \\ -2 & 1 & 1 \end{pmatrix} = (7,-2,-3)$

$$\Rightarrow \boxed{\varphi_2(x,y,z) = 7x - 2y - 3z}$$

φ_3: $(0,0,1)\begin{pmatrix} 1 & 0 & 0 \\ 7 & -2 & -3 \\ -2 & 1 & 1 \end{pmatrix} = (-2,1,1)$

$$\Rightarrow \boxed{\varphi_3(x,y,z) = -2x + y + z}$$

(ג) נמצא α, β, γ כך ש-

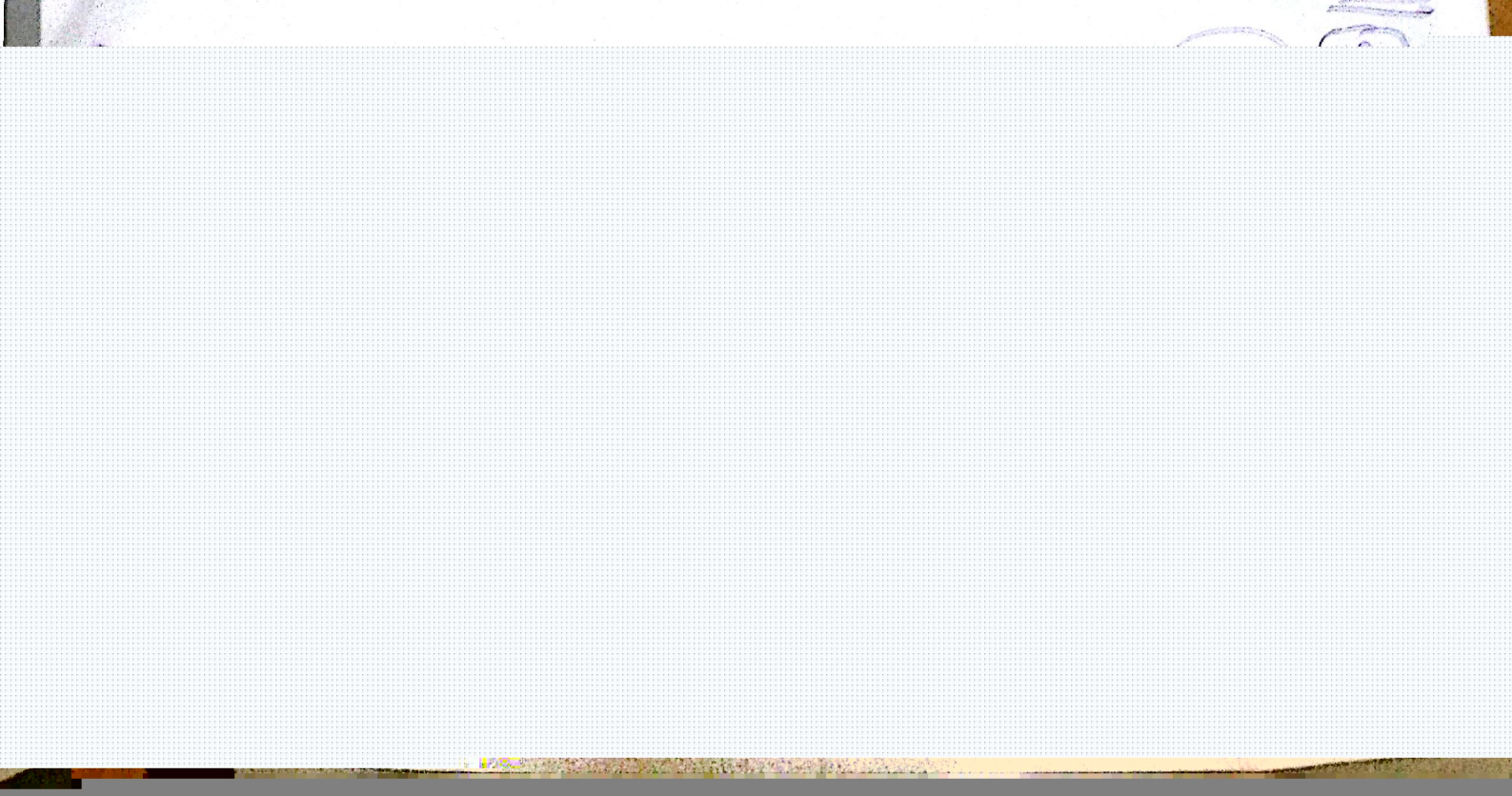
$$\alpha \cdot x + \beta(7x - 2y - 3z) + \gamma(-2x + y + z) = 9x - y - 3z$$

$$\left(\begin{array}{ccc|c} \alpha & \beta & \gamma & \\ 1 & 7 & -2 & 9 \\ 0 & -2 & 1 & -1 \\ 0 & -3 & 1 & -3 \end{array}\right) \dots\dots \begin{array}{l} \alpha = 1 \\ \beta = 2 \\ \gamma = 3 \end{array} \Rightarrow$$

מתקיימת שוויון נקבל

$$\boxed{[9x - y - 3z]_{B^*} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}}$$

Scanned by CamScanner



Scanned by CamScanner