

.1

$$a. f : Z \rightarrow Z \mid \forall n \in Z \quad f(n) = n^3 - n$$

$$f(1) = 1 \cdot (1^2 - 1) = 0 = (-1) \cdot ((-1)^2 - 1) = f(-1) \Rightarrow \text{ע"ק} \quad \text{ל} \quad f$$

$$2 \in Z : \neg \exists n \in Z \mid n^3 - n = 2 \Rightarrow \text{ע"ק} \quad \text{ל} \quad f$$

$$b. f : N \rightarrow N \mid \forall n \in N \quad f(n) = \begin{cases} n+1, & \text{זוגי} - \text{א} \quad n \\ n & , \text{זוגי} \quad n \end{cases}$$

$$f(1) = 2 = f(2) \Rightarrow \text{ע"ק} - \text{ל} \quad f$$

$$1 \in N : \neg \exists n \in N \mid f(n) = 1 \Rightarrow \text{ע"ק} \quad \text{ל} \quad f$$

$$c. \text{הקבוצה } A. \quad f : P(A) \rightarrow P(A) \mid \forall B \in P(A) \quad f(B) = A \setminus B$$

$$f(B) = f(C) : \text{ק"ק} \Rightarrow A \setminus B = A \setminus C \Rightarrow (x \in A \setminus B \Leftrightarrow x \in A \setminus C) \Rightarrow (x \in A \wedge B^c \Leftrightarrow x \in A \wedge C^c) \\ \Rightarrow (x \in A \wedge B \Leftrightarrow x \in A \wedge C) \Rightarrow (x \in B \Leftrightarrow x \in C) \Rightarrow B = C \Rightarrow \text{ע"ק} \quad f$$

$$\forall B \in P(A) \quad A \setminus B \in P(A) \Rightarrow A \setminus (A \setminus B) \subseteq A \Rightarrow A \setminus (A \setminus B) \in P(A) \wedge A \setminus (A \setminus B) = B \Rightarrow$$

$$\forall B \in P(A) \quad \exists C = A \setminus B \in P(A) : f(C) = A \setminus (C) = A \setminus (A \setminus B) = B \Rightarrow \text{ע"ק} \quad f$$

.2

.א

$$\forall n \in N, f(n) = n+1$$

$$f(n) = f(m) \Rightarrow n+1 = m+1 \Rightarrow n = m \Rightarrow \text{ע"ק} \quad \text{ק"ק}$$

$$\neg \exists n \in N : f(n) = 1 \Rightarrow \text{ע"ק} - \text{ל}$$

ב.

$$\forall n \in \mathbb{N}, f(n) = \begin{cases} 1, & n=1 \\ n-1, & \text{תתקן} \end{cases}$$

$$f(1) = 1 = f(2) \Rightarrow \text{ע"ת} - \text{ל}$$

$$\{1 \in \mathbb{N}, \exists 1 \in \mathbb{N} : f(1) = 1\}$$

$$\{\forall 1 \neq x \in \mathbb{N}, x+1 \in \mathbb{N} \Rightarrow \forall 1 \neq x \in \mathbb{N} \exists n \in \mathbb{N} : n = x+1 \Rightarrow \forall 1 \neq x \in \mathbb{N} \exists n \in \mathbb{N} : x = n-1 = f(n)\}$$

$$\Rightarrow \forall x \in \mathbb{N} \exists n \in \mathbb{N} : x = f(n) \Rightarrow \text{ע"ת}$$

3.

$$a. f : \mathbb{Z} \rightarrow \mathbb{Z} \mid \forall x \in \mathbb{Z} \quad f(x) = 2x+1$$

$$f(a) = f(b) \Rightarrow 2a+1 = 2b+1 \Rightarrow 2a = 2b \Rightarrow a = b \Rightarrow \text{ע"ת}$$

$$f(\mathbb{Z}) = \{y \in \mathbb{Z} : \exists x \in \mathbb{Z} \mid f(x) = y\} = \{y \in \mathbb{Z} : \exists x \in \mathbb{Z} \mid 2x+1 = y\} = \{1, \pm 3, \pm 5, \dots\} = \{y \in \mathbb{Z} : \text{זוגי} - \text{אי}\}$$

$$b. f : \mathbb{Q} \rightarrow \mathbb{Q} \mid \forall x \in \mathbb{Q} \quad f(x) = 2x+1$$

$$f(a) = f(b) \Rightarrow 2a+1 = 2b+1 \Rightarrow 2a = 2b \Rightarrow a = b \Rightarrow \text{ע"ת}$$

$$f(\mathbb{Z}) = \{y \in \mathbb{Q} : \exists x \in \mathbb{Q} \mid f(x) = y\} = \{y \in \mathbb{Q} : \exists x \in \mathbb{Q} \mid 2x+1 = y\} = \mathbb{Q}$$

↓

$$\overbrace{\forall y \in \mathbb{Q}, \frac{y-1}{2} \in \mathbb{Q} \Rightarrow \exists x \in \mathbb{Q} : x = \frac{y-1}{2} \Rightarrow \exists x \in \mathbb{Q} : y = 2x+1 = f(x) \Rightarrow \text{ע"ת}}$$

$$c. f : \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \rightarrow \mathbb{R} \mid \forall x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \quad f(x) = \sin(x)$$

$$f(a) = f(b) \Rightarrow \sin a = \sin b \Rightarrow a = b \Rightarrow \text{ע"ת}$$

$$a, b \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] : \text{ע"ת} \quad \downarrow$$

$$f\left(\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]\right) = \{y \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] : \exists x \in \mathbb{R} \mid f(x) = y\} = \{y \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] : \exists x \in \mathbb{R} \mid \sin x = y\} = [-1, 1]$$

***הערה לגבי א, ב: אלו ישרים ולכן תמיד חח"ע**

***הערה לגבי א: זו צורת כתיבה כללית למס' אי זוגי: $2x+1$, ולזוגי: $2x$, כאשר x מס' שלם)**

4. נתונות הפונ':

$$f, g : A = (-2, 7] \rightarrow R,$$

(קטע חצי סגור חצי פתוח כפי שהגדרנו בכיתה)

$$f = 2x - 4,$$

$$g = \frac{2x^2 - 8}{x + 2} = \frac{2(x^2 - 4)}{x + 2} = \frac{2(x - 2)(x + 2)}{x + 2} = 2(x - 2) = 2x - 4 = f(x)$$

↓

כי המכנה שונה מאפס בשל הגדרת התחום (-2, מאפס המכנה, לא נמצא בו) ולכן ניתן לצמצם את $x+2$

5. נתונה הפונ': $f : R \rightarrow R \mid \forall x \in R \quad f(x) = x^2$

a. $f(A) \mid A = (0, 1] \cup [2, 3)$

$$f((0, 1] \cup [2, 3)) = (0, 1] \cup [4, 9)$$

b. $f(A) \mid A = (-2, -1) \cup (1, 2)$

$$f((-2, -1) \cup (1, 2)) = (1, 4)$$

$$a. f: Z \rightarrow Z \mid \forall x \in Z \quad f(x) = 2x$$

$$f(a) = f(b) \Rightarrow 2a = 2b \Rightarrow a = b \Rightarrow \text{y'' } \pi\pi$$

$$1 \in Z : \neg \exists x \in Z : f(x) = 2x = 1 \Rightarrow \text{ly } \text{xl}$$

$$b. f: R \rightarrow R \mid \forall x \in R \quad f(x) = 2x$$

$$f(a) = f(b) \Rightarrow 2a = 2b \Rightarrow a = b \Rightarrow \text{y'' } \pi\pi$$

$$\forall y \in R \quad \frac{y}{2} \in R \Rightarrow \forall y \in R \quad \exists x \in R : x = \frac{y}{2} \Rightarrow \forall y \in R \quad \exists x \in R : y = 2x = f(x) \Rightarrow \text{ly}$$

$$f^{-1}(x) = \frac{x}{2}$$

$$c. f: R \rightarrow R \mid \forall x \in R \quad f(x) = \frac{1}{1+x^2}$$

$$f(2) = \frac{1}{1+2^2} = \frac{1}{1+(-2)^2} = f(-2) \Rightarrow \text{y'' } \pi\pi \quad \text{xl}$$

$$\sqrt{\frac{1}{2}-1} \notin R, 2 \in R \Rightarrow 2 \in R : \neg \exists x \in R : \frac{1}{1+x^2} = 2 \Rightarrow \text{ly } \text{xl}$$

$$d. f: R^+ \cup \{0\} \rightarrow (0,1] \mid \forall x \in R^+ \cup \{0\} \quad f(x) = \frac{1}{1+x^2}$$

$$f(a) = f(b) \Rightarrow \frac{1}{1+a^2} = \frac{1}{1+b^2} \Rightarrow 1+a^2 = 1+b^2 \Rightarrow a^2 = b^2 \Rightarrow a = b \Rightarrow \text{y'' } \pi\pi$$

$$\forall y \in (0,1] \quad \sqrt{\frac{1}{y}-1} \in R^+ \cup \{0\} \Rightarrow \forall y \in (0,1] \quad \exists x \in R^+ \cup \{0\} : x = \sqrt{\frac{1}{y}-1} \Rightarrow$$

$$\forall y \in (0,1] \quad \exists x \in R^+ \cup \{0\} : f(x) = \frac{1}{1+x^2} = y \Rightarrow \text{ly}$$

$$f^{-1}(x) = \sqrt{\frac{1}{x}-1}$$

f:

$$\begin{aligned}
 & f \circ g \circ f \text{ הפיכה} \\
 & \Downarrow \\
 & f \circ (g \circ f) \text{ חח"ע"ע} + \text{על} \\
 & \quad \Downarrow \quad \Downarrow \\
 & \underline{\underline{f \text{ על}}} \quad \underline{\underline{g \text{ חח"ע"ע} \circ f}} \\
 & \quad \quad \quad \Downarrow \\
 & \quad \quad \quad \underline{\underline{\underline{f \text{ חח"ע"ע}}}}
 \end{aligned}$$

ובסה"כ f הפיכה**g:**

$$\begin{aligned}
 & f \circ g \circ f \text{ הפיכה} \\
 & \Downarrow \\
 & (f \circ g) \circ f \text{ חח"ע"ע} + \text{על} \\
 & \quad \Downarrow \quad \Downarrow \\
 & \underline{\underline{f \text{ על} \circ g}} \quad \underline{\underline{f \text{ חח"ע"ע}}}} \\
 & \quad \quad \quad \Downarrow \\
 & \quad \quad \quad \underline{\underline{\underline{g \text{ על}: ג2 מסעיף}}}
 \end{aligned}$$

כמו - כן :

$$\begin{aligned}
 & g(a) = g(b) : \text{נניח} \Rightarrow (\exists x, y : f(x) = a, f(y) = b : f \text{ על"ע"ע} \leftarrow) \quad g(f(x)) = g(f(y)) \Rightarrow \\
 & (\leftarrow \text{מלמעלה} : g \text{ חח"ע"ע} \circ f) \quad x = y \Rightarrow (f \text{ פונ}') \quad a = f(x) = f(y) = b \Rightarrow \underline{\underline{\underline{g \text{ חח"ע"ע}}}}
 \end{aligned}$$

ובסה"כ g הפיכה