

$$\int \sin^2(x) \cos^2(x) dx$$

$$\sin^2(x) = (1 - \cos^2(x))$$

$$\int (1 - \cos^2(x))$$

(1)

אינטגרציה של פונקציות רציונליות

$$\int \frac{dx}{x^2+3x-4}$$

(2)

$$0 < 9 - 21 \cdot (-4) = 25 = 5^2 \text{ : } \Delta > 0$$

$$(x-4)(x+1) \text{ : } \Delta > 0 \text{ : } \text{פירוק}$$

$$\frac{A}{x-4} + \frac{B}{x+1} = \frac{1}{x^2+3x-4}$$

$$A(x+1) + B(x-4) = 1$$

$$(A+B)x + B(-4) = 1 \quad (x=-1 \text{ : } \Delta)$$

$$5A = 1 \Rightarrow A = \frac{1}{5}$$

$$5A = 1 \Rightarrow A = \frac{1}{5} \quad (x=4 \text{ : } \Delta)$$

$$\int \frac{dx}{x^2+3x-4} = \frac{1}{5} \int \frac{dx}{x-4} - \frac{1}{5} \int \frac{dx}{x+1} =$$

$$= \frac{1}{5} (\ln|x-4| - \ln|x+1|) + C =$$

$$= \frac{1}{5} \ln \left| \frac{x-4}{x+1} \right| + C$$

$$\int \frac{x}{x^2-5x+6} dx$$

(3)

$$0 < 25 - 4 \cdot 6 = 1 = 1^2 \text{ : } \Delta > 0$$

$$x^2-5x+6 = (x-2)(x-3)$$

2

$$\frac{A}{x-2} + \frac{B}{x-3} = \frac{x}{(x-2)(x-3)}$$

$$A(x-3) + B(x-2) = x$$

$$B \cdot 1 = 3 \Rightarrow B = 3 \quad : x=3 : 0?)$$

$$-A = 2 \Rightarrow A = -2 \quad x=2 : 0?)$$

: $k(x) = \frac{1}{(x-2)(x-3)}$

$$\frac{-2}{x-2} + \frac{3}{x-3}$$

↓

$$\int \frac{x}{x^2-5x+6} dx = \int \frac{-2}{x-2} dx + \int \frac{3}{x-3} dx =$$

$$= -2 \ln|x-2| + 3 \ln|x-3| + c$$

$$\int \frac{11x+17}{2x^2+7x-4}$$

3

: $k(x) = \frac{11x+17}{(2x-1)(x+4)}$

$$(2x-1)(x+4)$$

$$\frac{A}{2x-1} + \frac{B}{x+4} = \frac{11x+17}{2x^2+7x-4}$$

$$A(x+4) + B(2x-1) = 11x+17$$

$$B(-9) = -44+17 = 27 \quad : x=-4$$

↓

$$B = -3$$

$$A\left(\frac{1}{2}+4\right) = \frac{11}{2}+17 \quad x=\frac{1}{2}$$

$x=\frac{1}{2}$

$$A \frac{9}{2} = \frac{11+34}{2} \Rightarrow 45 \Rightarrow A = 5$$

$$\frac{5}{2x-1} + \frac{3}{x+4}$$

$$\int \frac{11x+17}{2x^2+7x-4} dx = \int \frac{5 dx}{2x-1} + \int \frac{3}{x+4} dx = \frac{5}{2} \ln|2x-1| + 3 \ln|x+4| + c$$

$$\int \frac{dx}{(x-1)(x+3)(x-3)}$$

(4)

(3)

$$\frac{A}{x-1} + \frac{B}{x+3} + \frac{C}{x-3} = \frac{1}{(\quad)}$$

:D/D

$$A(x+3)(x-3) + B(x-1)(x-3) + C(x-1)(x+3) = 1$$

$$A(4)(-2) = -8A = 1 \quad A = -\frac{1}{8} \quad \text{: } X=1 \text{ D?}$$

$$C \cdot 2 \cdot 6 = 1 \Rightarrow C = \frac{1}{12} \quad \text{: } X=3 \text{ D?}$$

$$B(-4)(-6) = 24B = 1 \quad \text{: } X=-3 \text{ D?}$$
$$B = \frac{1}{24}$$

$$-\frac{1}{8} \cdot \frac{1}{x-1} + \frac{1}{12} \frac{1}{x+3} + \frac{1}{24} \cdot \frac{1}{x-3}$$

$$-\frac{1}{8} \int \frac{1}{x-1} dx + \frac{1}{12} \int \frac{1}{x+3} dx + \frac{1}{24} \int \frac{1}{x-3} dx =$$

$$= -\frac{1}{8} \ln|x-1| + \frac{1}{12} \ln|x+3| + \frac{1}{24} \ln|x-3|$$

$$\int \frac{2x^2 - 9x - 9}{x^3 - 9x} dx \quad \int \frac{2x^2 - 9x - 9}{x(x^2 - 3)(x+3)} dx \quad (5)$$

$$\frac{A}{x} + \frac{B}{x-3} + \frac{C}{x+3} = \frac{2x^2 - 9x - 9}{(\quad)}$$

$$A(x-3)(x+3) + Bx(x+3) + Cx(x-3) = 2x^2 - 9x - 9$$

$$A(-9) = -9 \Rightarrow A = 1 \quad \text{: } X=0$$

$$B \cdot 3 \cdot 6 = 2 \cdot 9 - 9 \cdot 3 - 9 = 9(2 - 3 - 1) = 9 \cdot (-2) = -18 \quad X=3$$
$$B = -1$$

$$C(-3)(-6) = 2 \cdot 9 + 9 \cdot 3 - 9 = 9(2 + 3 - 1) = 9 \cdot 4 = 36 \quad \text{: } X=-3$$
$$C = +2$$

$$\frac{1}{x} - \frac{1}{x-3} + \frac{2}{x+3}$$

(4) ~~(5)~~

$$\int \frac{2x^2 - 9x - 9}{x^3 - 9x} dx = \int \left(\frac{1}{x} - \frac{1}{x-3} + \frac{2}{x+3} \right) dx =$$

$$= \ln|x| - \ln|x-3| + 2\ln|x+3| + c =$$

$$= \ln \left| \frac{x(x+3)^2}{x-3} \right| + c$$

$$\int \frac{3x^2 - 10}{x^2 - 4x + 4} dx = 3 \int \frac{x^2 - 4 + 4}{(x-2)^2} dx - \int \frac{10}{(x-2)^2} dx = \quad (6)$$

$$= 3 \int \frac{x+2}{x-2} dx + \int \frac{2 dx}{(x-2)^2} = 3 \int \frac{x-2+4}{x-2} dx +$$

$$+ 2 \left(-\frac{1}{x-2} \right) = 3x + 12\ln|x-2| - \frac{2}{x-2} + c$$

$$\int \frac{x^3}{x^2 - 3x + 2} dx = \int \frac{(x+3)(x^2 - 3x + 2) + 7x - 2}{(x^2 - 3x + 2)} dx = \quad (7)$$

$$\frac{x+3}{x^3} \overline{x^2 - 3x + 2} \left. \begin{array}{l} x^2 - 3x + 2 \\ -x^3 - 3x^2 + 2x \\ \hline 3x^2 - 2x \\ -3x^2 - 9x + 6 \\ \hline 7x - 6 \end{array} \right\} = \int (x+3) dx + \int \frac{7x-6}{x^2-3x+2} dx = \frac{x^2}{2} + 3x + \int \frac{7x-6}{(x-2)(x-1)} dx$$

$$\frac{A}{x-2} + \frac{B}{x-1} = \frac{7x-6}{(x-2)(x-1)}$$

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$$A = 8 \quad B = -1$$

$$\int \frac{x^3}{x^2 - 3x + 2} dx = \frac{x^2}{2} + 3x + \int \frac{8}{x-2} dx - \int \frac{1}{x-1} dx =$$

$$= \frac{x^2}{2} + 3x + 8\ln|x-2| - \ln|x-1| + c$$

$$\int \frac{x^5 + 2x^2 + 1}{x^3 - x} dx \quad (8)$$

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~~$\frac{x^2+1}{x^3-x}$~~

$$\frac{x^2+1}{x^3-x} = \frac{x^2+1}{x^3-x} + \frac{2x^2+x+1}{x^3-x}$$

$$\frac{x^3+2x^2+1}{x^3-x}$$

$$\frac{x^3+2x^2+1 - x^3+x}{x^3-x}$$

$$\frac{2x^2+x+1}{x^3-x}$$

$$\int \frac{x^5+2x^2+1}{x^3-x} dx = \int \frac{(x^2+1)(x^3-x) + 2x^2+x+1}{x^3-x} dx =$$

$$= \int (x^2+1) dx + \int \frac{2x^2+x+1}{x^3-x} dx =$$

$$= \frac{x^3}{3} + x + \int \frac{2x^2+x+1}{x(x-1)(x+1)} dx$$

$$\frac{A}{x} + \frac{B}{x-1} + \frac{C}{x+1} = \frac{2x^2+x+1}{x^3-x}$$

$$A(x-1)(x+1) + Bx(x+1) + Cx(x-1) = 2x^2+x+1$$

$$A = -1 \quad : x=0$$

$$B = 2 \quad : x=1$$

$$C = -1 \quad : x=-1$$

$$\int \frac{x^5+2x^2+1}{x^3-x} dx = \frac{x^3}{3} + x + \int -\frac{1}{x} dx + \int \frac{2}{x-1} dx +$$

$$+ \int -\frac{1}{x+1} dx = \frac{x^3}{3} + x - \ln|x| + 2\ln|x-1| -$$

$$- \ln|x+1| = \ln \left| \frac{(x-1)^2}{x(x+1)} \right| + c + \frac{x^3}{3} + x$$

∴ c=0

~~$\frac{x^3}{3}$~~

$$\int \frac{2x^2+3}{x(x-1)^2} dx$$

19 ⑥ ⑦

1710

$$\frac{A}{x} + \frac{B}{x-1} + \frac{C}{(x-1)^2} = \frac{2x^2+3}{x(x-1)^2}$$

$$A(x-1)^2 + Bx(x-1) + Cx = 2x^2+3$$

$$A(x^2-2x+1) + B(x^2-x) + Cx = (A+B)x^2 + (C-B-2A)x + A = 2x^2+3$$

$$\boxed{A=3} \quad A+B=2 \Rightarrow \boxed{B=-1}$$

$$C+1-6=0 \Rightarrow \boxed{C=5}$$

$$\int \frac{3}{x} dx + \int \frac{-1}{x-1} dx + \int \frac{5}{(x-1)^2} dx =$$

$$= 3 \ln|x| - \ln|x-1| - \frac{5}{x-1} + C$$

$$\int \frac{x^2+x-16}{(x+1)(x-3)^2} dx$$

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$$\frac{A}{x+1} + \frac{B}{x-3} + \frac{C}{(x-3)^2} = x^2+x-16$$

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$$A(x-3)^2 + B(x+1)(x-3) + C(x+1) = x^2+x-16$$

$$4C = 9+3-16 = -4$$

$x=3$

$$\boxed{C=-1}$$

$$16A = 1-1-16 \Rightarrow \boxed{A=-1}$$

$x=-1$

$$-9+B(-3)-1 = -16$$

$x=0$

$$-3B = -6 \Rightarrow \boxed{B=2}$$

$$\int \frac{x^2+x-16}{(x+1)(x-3)^2} dx = \int \frac{-1}{x+1} dx + \int \frac{2}{x-3} dx + \int \frac{-1}{(x-3)^2} dx =$$

$$= -\ln|x+1| + 2\ln|x-3| + \frac{1}{x-3} + C$$

$$\int \frac{2x^2-1}{(4x-1)(x^2+1)} dx$$

A11

(7)

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$$\frac{A}{4x-1} + \frac{Bx+C}{x^2+1} = \frac{2x^2-1}{(4x-1)(x^2+1)}$$

$$A(x^2+1) + (Bx+C)(4x-1) = 2x^2-1$$

$$Ax^2 + A + 4Bx^2 + 4Cx - Bx - C = (A+4B)x^2 + (4C-B)x + A-C = 2x^2 - 1$$

$$A - C = -1 \Rightarrow A = C - 1$$

$$4C - B = 0 \Rightarrow B = 4C$$

$$-1 + C + 16C = 17C - 1 = 2 \Rightarrow$$

$$\Rightarrow 17C = 3 \Rightarrow C = \frac{3}{17}$$

$$B = \frac{12}{17} \quad A = \frac{3}{17} - 1 = -\frac{14}{17}$$

$$\int \frac{2x^2-1}{(4x-1)(x^2+1)} dx = \int -\frac{14}{17} \frac{1 dx}{4x-1} + \int \frac{12}{17} \frac{x}{x^2+1} dx +$$

$$+ \int \frac{3}{17} \cdot \frac{1}{x^2+1} dx = -\frac{7}{34} \ln|4x-1| + \frac{6}{17} \ln(x^2+1) + \frac{3}{17} \operatorname{arctg} x +$$

+c

$$\int \frac{x^3 - 3x^2 + 2x - 3}{x^2 + 1} dx = \int (x-3) dx + \int \frac{x}{x^2+1} dx = \frac{1}{2} \ln 2$$

$$= \frac{x^2}{2} - 3x + \frac{1}{2} \ln(x^2+1) + c$$

$$\begin{array}{r} x-3 \\ \hline x^3-3x^2+2x-3 \end{array} \bigg| x^2+1$$

$$\begin{array}{r} x^3+x \\ \hline \end{array}$$

$$-3x^2+x-3$$

$$-3x^2-3$$

x