

$$11) \int \frac{x^3 + 3x^2 + 5x + 7}{x^2 + 2} dx = \int (x+3) dx + \int \frac{3x+1}{x^2+2} dx =$$

$$= \frac{x^2}{2} + 3x + \frac{3}{2} \int \frac{2x}{x^2+2} dx + \int \frac{dx}{x^2+2} =$$

$$= \frac{x^2}{2} + 3x + \ln(x^2+2) + \frac{1}{2} \arctan \frac{x}{\sqrt{2}} + C$$

$$\begin{array}{r} x+3 \\ \overline{x^3+3x^2+5x+7} \quad x^2+2 \\ -x^3 \quad \quad \quad +2x \\ \hline -3x^2+3x+7 \\ -3x^2 \quad \quad \quad +6 \\ \hline 3x+1 \end{array}$$

$$12) \int \frac{x^5}{(x^3+1)(x^3+8)} dx = \left[\begin{array}{l} t = x^3 + 1 \\ dt = 3x dx \\ t+7 = x^3+8 \\ t-1 = x^3 \end{array} \right] = \int \frac{(t-1)}{3t(t+7)} dt$$

$$\frac{t-1}{t(t+7)} = \frac{A}{t} + \frac{B}{t+7}$$

$$t-1 = A(t+7) + Bt$$

$$23) \Rightarrow t=0 \Rightarrow -1 = 7A \Rightarrow A = -\frac{1}{7}$$

$$t=-7 \Rightarrow -8 = -7B \Rightarrow B = \frac{8}{7}$$

$$\Rightarrow \int \frac{x^5}{(x^3+1)(x^3+8)} dx = \frac{1}{3} \left(-\frac{1}{7} \right) \int \frac{dt}{t} + \frac{1}{3} \cdot \frac{8}{7} \int \frac{dt}{t+7} =$$

$$= -\frac{1}{21} \ln|t| + \frac{8}{21} \ln|t+7| + C$$

$$= -\frac{1}{21} \ln|x^3+1| + \frac{8}{21} \ln|x^3+8| + C$$

$$13) \int \cos 3x \cos 2x dx = \int \frac{1}{2} (\cos 5x + \cos x) dx =$$

$$= \frac{1}{10} \sin 5x + \frac{1}{2} \sin x + C$$

$$\cos \alpha \cos \beta = \frac{1}{2} (\cos(\alpha+\beta) + \cos(\alpha-\beta))$$

$$\begin{aligned} 14) \int \cos x \cos 2x \cos 4x \, dx &= \int \frac{1}{2} (\cos 3x + \cos x) \cos 4x \, dx = \\ &= \int \frac{1}{4} (\cos 7x + \cos x + \cos 5x + \cos 3x) \, dx = \\ &= \frac{1}{28} \sin 7x + \frac{1}{4} \sin x + \frac{1}{20} \sin 5x + \frac{1}{12} \sin 3x + C \end{aligned}$$

$$15) \int \tan^2 x \, dx = \int \frac{1}{\cos^2 x} - 1 \, dx = \tan x - x + C$$