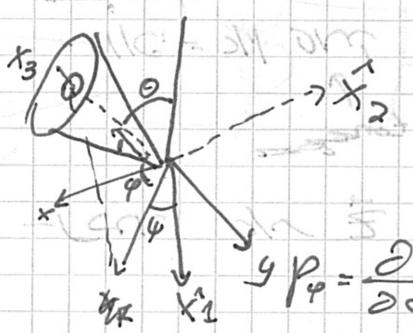
הרצאה 17:



$$\tilde{L} = \frac{I_1 + ml^2}{2}\left(\dot{\theta}^2 + \dot{\varphi}^2 \sin^2\theta\right) + \frac{I_3}{2}\left(\dot{\psi} + \dot{\varphi}\cos\theta\right)^2 - mgl\cos\theta$$

$$P_\varphi = \frac{\partial \tilde{L}}{\partial \dot{\varphi}} = L_z = \text{const}$$

$$L_z = \left(I_1 + ml^2 \sin^2\theta + I_3\cos^2\theta\right)\dot{\varphi} + I_3\dot{\psi}\cos\theta = L_z$$

$$P_\psi = \frac{\partial L}{\partial \dot{\psi}} = L_3$$

$$\frac{d}{dt}\frac{\partial \tilde{L}}{\partial \dot{\theta}} - \frac{\partial L}{\partial \theta}$$

$$\dot{\varphi} = \frac{L_z - L_3\cos\theta}{(I_1 + ml^2)\sin^2\theta} \qquad \dot{\psi} = \frac{L_3}{I_3} - \cos\theta\,\frac{L_z - L_3\cos\theta}{(I_1 + ml^2)\sin^2\theta}$$

$$E = \left(\frac{I_1 + ml^2}{2}\right)\left(\dot{\theta}^2 + \dot{\varphi}^2\sin^2\theta\right) + \frac{I_3}{2}\left(\dot{\psi} + \dot{\varphi}\cos\theta\right)^2 + mgl\cos\theta$$

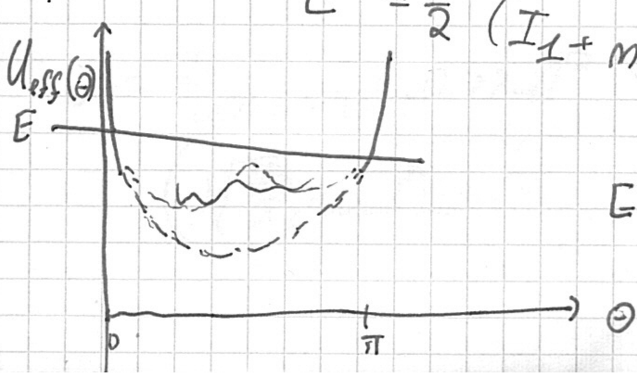
$$E = \left(\frac{I_1 + ml^2}{2}\right)\dot{\theta}^2 + \boxed{\text{תלוי ב-}\theta\text{ בלבד}}$$

$$E = \frac{1}{2}\left(I_1 + ml^2\right)\dot{\theta}^2 + \frac{(L_z - L_3\cos\theta)^2}{2(I_1 + ml^2)\sin^2\theta} + mgl\cos\theta + \frac{L_3^2}{2I_3}$$

$$\underbrace{E - \frac{L_3^2}{2I_3} - mgl\cos\theta}_{E'} = \frac{1}{2}(I_1 + ml^2)\dot{\theta}^2 + \frac{(L_z - L_3\cos\theta)^2}{2(I_1 + ml^2)\sin^2\theta} - mgl(1 - \cos\theta)$$

$$U_{eff}(\theta) = \frac{(L_z - L_3\cos\theta)^2}{2(I_1 + ml^2)\sin^2\theta} - mgl(1 - \cos\theta)$$

$$E' = \frac{1}{2}(I_1 + ml^2)\dot{\theta}^2 + U_{eff}(\theta)$$



$$E' = \frac{1}{2}(I_1 + ml^2)\dot{\theta}^2 + \frac{(L_z - L_3\cos\theta)^2}{2(I_1 + ml^2)\sin^2\theta} - mgl(1 - \cos\theta)$$

$$\sin^2\theta\, E' = \frac{1}{2}(I_1+ml^2)\underbrace{(\dot\theta\sin\theta)^2}_{-\frac{d\cos\theta}{dt}} + \frac{(L_z-L_3\cos\theta)^2}{2(I_1+ml^2)} \cancel{-mgl}$$

$$-mgl(1-\cos\theta)\sin^2\theta$$

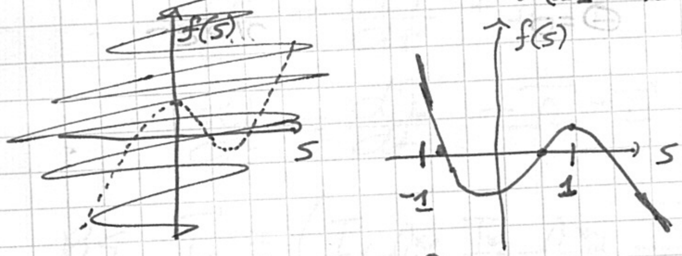
$$\cos\theta = S$$

$$(1-S^2)E' = \frac{1}{2}(I_1+ml^2)\dot S^2 + \frac{(L_z-L_3S)^2}{2(I_1+ml^2)} - mgl(1-S)(1-S^2)$$

$$\dot S = 0 = \dot\theta\,\cancel{\cos}\sin\theta$$

$$(1-S^2)E' = \frac{(L_z-L_3S)^2}{2(I_1+ml^2)} - mgl(1-S)(1-S^2)$$

$$f(S) = -(1-S^2)E' + \frac{(L_z-L_3S)^2}{2(I_1+ml^2)} - mgl(1-S)(1-S^2)$$

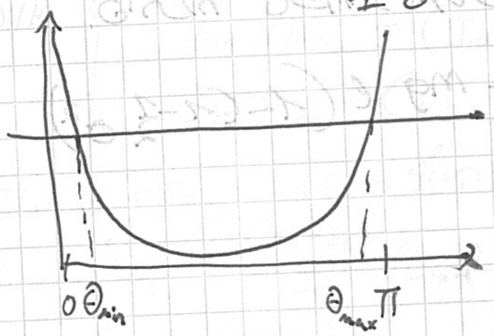


סביר $S=1$:

$$f(1) = \frac{(L_z-L_3)^2}{2(I_1+ml^2)}$$

ובגלל זה יש שני חיתוכים בין 1 ל 1-

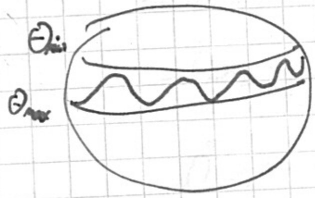
ולכן



$$\dot\varphi = \frac{L_z-L_3\cos\theta}{(I_1+ml^2)\sin^2\theta}$$

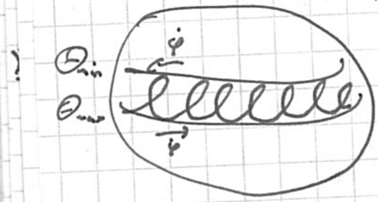
I) $\dot\varphi > 0 \quad \forall\theta$

אם $L_z > L_3$

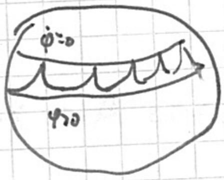


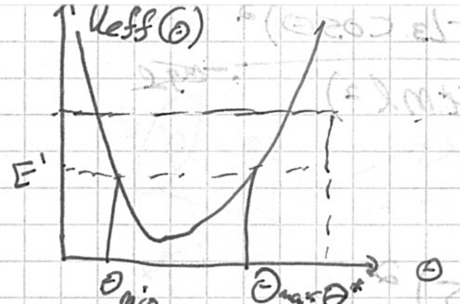
II) $L_z - L_3\cos\theta$ מחליף סימן

$$L_z < L_3$$



III) $L_z - L_3\cos\theta_{min} = 0$

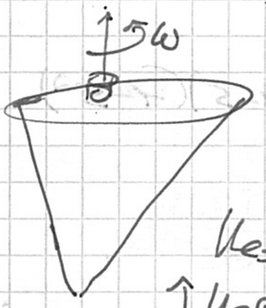




קריטי

פה הגוף יעבור ויגיע בקצה..

<u>שאלה:</u> מה הוא תנאי ליציבות של סביבון סימטרי סביב $\theta=0$.

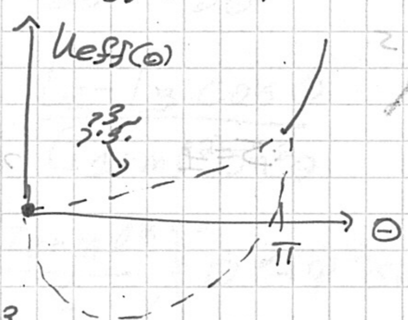


$$E-\frac{L_3^2}{2I_3}-mgl=\frac{(L_z-L_3\cos\theta)^2}{2(I+ml^2)\sin^2\theta}-mgl(1-\cos\theta)$$

עבור

$\theta=0$

$$U_{eff}(\theta=0)=0$$



פיתוח לזוויות קטנות של $U_{eff}(\theta)$? $\sim\theta^2$ יציב.

$$U_{eff}(\theta)\underset{\theta\ll1}{\approx}\frac{(L_3-L_3(1-\frac{1}{2}\theta^2))^2}{2(I_1+ml^2)\theta^2}-mgl\left(1-(1-\frac{1}{2}\theta^2)\right)$$

$$=\frac{(L_3-L_3+\frac{1}{2}L_3\theta^2)^2}{2(I_1+ml^2)\theta^2}-mgl\cdot\frac{1}{2}\theta^2$$

$$\frac{1}{2}\left[\frac{1}{4}\frac{L_3^2}{(I_1+ml^2)}-mgl\right]\theta^2$$

$$\boxed{\frac{1}{4}\frac{L_3^2}{(I_1+ml^2)}>mgl}$$

מספיק

תנאי ליציבות.