19.3.19

פתרון תרגיל אינטגרלים 3

(1) $g'=1$ $f'=e^x$

$g=x$ $f=e^x$

$$\int x\cdot e^x\,dx$$

$$= xe^x-\int e^x\,dx = xe^x-e^x+c$$

(2) $g'=\frac{1}{x}$ $f'=x^4$

$g=\ln(x)$ $f=\frac{x^5}{5}$

$$\int x^4\ln(x)\,dx$$

$$=\frac{x^5\ln(x)}{5}-\int\frac{x^4}{5}\,dx$$

$$=\frac{x^5\ln(x)}{5}-\frac{x^5}{25}+c$$

(3) $g'=1$ $f'(x)=\sin(x)$

$g=x$ $f=-\cos(x)$

$$\int x\sin(x)\,dx$$

$$=-x\cos(x)-\int-\cos(x)\,dx$$

$$=-x\cos(x)+\sin(x)+c$$

(4) $g'=\frac{-\sin(\ln(x))}{x}$ $f'=1$

$g=\cos(\ln x)$ $f=x$

$$\int\cos(\ln(x))\,dx$$

$$= x\cos(\ln(x))+\int x\frac{\sin(\ln(x))}{x}\,dx$$

$g'=\frac{+\cos(\ln(x))}{x}$ $f'=1$

$g=\sin(\ln(x))$ $f=x$

$$= x\cos(\ln(x))+x\sin(\ln(x))-\int\cos(\ln(x))\,dx$$

$$\int\cos(\ln(x))=\frac{1}{2}x\cos(\ln(x))+\frac{1}{2}x\sin(\ln(x))+c$$

$\int e^{2x}\sin(4x)\,dx$ $\quad f'=e^{2x}$ $\quad g'=4\cos(4x)$ (5)

$f=\frac{e^{2x}}{2}$ $\quad g=\sin(4x)$

$= \frac{1}{2}e^{2x}\sin(4x) - \int 2e^{2x}\cos(4x)\,dx$

$= \frac{1}{2}e^{2x}\sin(4x) - \frac{2}{2}e^{2x}\cos(4x) - 2\int 2e^{2x}\sin(4x)\,dx$ $\quad f'=e^{2x}$ $\quad g'=-4\sin(4x)$

$+5\int e^{2x}\sin(4x)\,dx = \frac{1}{2}e^{2x}[\sin(4x)-2\cos(4x)]$ $\quad f=\frac{e^{2x}}{2}$ $\quad g=\cos(4x)$

$= \frac{1}{10}e^{2x}(\sin(4x)-2\cos(4x)] + C$

(6)

$\int x\ln(x^2+1)\,dx$ $\quad f'=x$ $\quad g'=\frac{2x}{x^2+1}$

$f=\frac{x^2}{2}$ $\quad g=\ln(x^2+1)$

$= \frac{x^2}{2}\ln(x^2+1) - \int \frac{x^3}{x^2+1}\,dx$

$\int x - \frac{x}{x^2+1}$ $\rightarrow \frac{g'}{2}$ $\quad \frac{x^3}{x^2+1} = $ $x^3 \div (x^2+1)$: x; $-(x^3+x)$; $-x$

$= \frac{x^2}{2}\ln(x^2+1) - \frac{x^2}{2} + \frac{\ln(x^2+1)}{2} + C$ $\quad \Rightarrow x - \frac{x}{x^2+1}$

$\int x^2\,\mathrm{arctg}(x)\,dx$ $\quad f'=x^2$ $\quad g'=\frac{1}{1+x^2}$ (7)

(6) $\Rightarrow$

$= \frac{1}{3}x^3\mathrm{arctg}(x) - \frac{1}{3}\int \frac{x^3}{1+x^2}\,dx$ $\quad f=\frac{x^3}{3}$ $\quad g=\mathrm{arctg}(x)$

$= \frac{1}{3}x^3\mathrm{arctg}(x) - \frac{x^2}{6} + \frac{\ln(x^2+1)}{6} + C$

(8)

$$\int \frac{1}{(x^2+1)(x^2-1)}\,dx$$

$$\frac{A}{x^2+1} + \frac{B}{x^2-1} = \frac{1}{(x^2-1)(x^2+1)}$$

$$\frac{A(x^2-1) + B(x^2+1)}{(x^2+1)(x^2-1)} = \frac{1}{(x^2+1)(x^2-1)}$$

$$\Rightarrow Ax^2 - A + Bx^2 + B = 1$$

$$\Rightarrow A = -B \qquad A = B - 1$$

$$B - 1 = -B$$

$$2B = 1$$

$$B = \frac{1}{2} \Rightarrow A = -\frac{1}{2}$$

$$\frac{1}{2}\int \frac{-1}{x^2+1} + \frac{1}{x^2-1}\,dx$$

$$\frac{A}{(x+1)} + \frac{B}{(x-1)} = \frac{1}{(x+1)(x-1)}$$

$$Ax - A + Bx + B = 1$$

$$A = -B \qquad A = B - 1$$

$$\Rightarrow A = -\frac{1}{2} \qquad B = \frac{1}{2}$$

$$\frac{1}{2}\int \frac{-1}{x^2+1}\,dx + \frac{1}{2}\int \frac{-1}{x+1} + \frac{1}{x-1}\,dx$$

$$-\frac{1}{2}\operatorname{arctg}(x) + \frac{1}{2}\ln(|x+1|) + \frac{1}{2}\ln(|x-1|) + C$$

(א)

$$\int \frac{1}{x^2-1}\,dx$$

לפ' (8 e)

$$= -\frac{1}{2}\ln(|x+1|) + \frac{1}{2}\ln(|x-1|) + C$$

(ב)

$$\int \frac{x}{x^2-4}\,dx$$

$$\frac{A}{x-2} + \frac{B}{x+2} = \frac{x}{(x-2)(x+2)}$$

$$Ax + 2A + Bx - 2B = x$$

$$A + B = 1 \qquad A = B$$

$$\Rightarrow \quad A = \frac{1}{2} = B$$

$$= \frac{1}{2}\int \frac{1}{x-2} + \frac{1}{x+2}\,dx$$

$$\frac{1}{2}\ln(|x-2|) + \frac{1}{2}\ln(|x+2|) + C$$

(ג)

$$\int \frac{1}{x^2+4}\,dx$$

רוצים להגיע
למכנה מהצורה
$f^2(x) + 1$

$$\arctan(f(x))' \rightarrow \frac{f'(x)}{f^2(x)+1}$$

לכן

$$\int \frac{1 \cdot \frac{1}{4}}{(x^2+4)\frac{1}{4}}\,dx = \int \frac{\frac{1}{4}}{\frac{x^2}{4}+1}\,dx$$

$$\frac{1}{2}\int \frac{\frac{1}{2}}{\left(\frac{x}{2}\right)^2+1}\,dx \Rightarrow \frac{1}{2}\arctan\left(\frac{x}{2}\right)$$

(7)

$$\int \frac{x+4}{x^2-2x+5}\,dx$$

$$\int \frac{\frac{1}{2}(2x-2)}{x^2-2x+5} + \frac{5}{x^2-2x+5}\,dx$$

$$\frac{f'(x)}{f^2(x)+1}$$

$$\frac{1}{2}\int \frac{2x-2}{x^2-2x+5}\,dx + 5\int \frac{1}{x^2-2x+5}\,dx$$

$$5\int \frac{1}{(x-1)^2+4}\,dx = 5\int \frac{1\cdot\frac{1}{4}}{\frac{(x-1)^2}{4}+\frac{4}{4}}\,dx$$

$$(x-1)^2 = x^2-2x+1$$

$$= 5\int \frac{\frac{1}{4}}{\left(\frac{x-1}{2}\right)^2+1}\,dx = \frac{5}{2}\int \frac{\frac{1}{2}}{\left(\frac{x-1}{2}\right)^2+1}\,dx$$

$$\frac{1}{2}\int \frac{2x-2}{x^2-2x+5}\,dx + \frac{5}{2}\int \frac{\frac{1}{2}}{\left(\frac{x-1}{2}\right)^2+1}\,dx$$

$$\frac{1}{2}\ln\left(x^2-2x+5\right) + \frac{5}{2}\operatorname{arctg}\left(\frac{x-1}{2}\right)+C$$

$$\int \frac{4}{x-1}\,dx$$ (2)

$$4\int \frac{1}{x-1}\,dx \rightarrow 4\ln(|x-1|)+C$$

$$\int \frac{3x+2}{x-1}\,dx = \int 3+\frac{5}{x-1}\,dx$$ (9)

$$3x+5\ln(x-1)+C$$

$$\int \frac{2x+1}{7x+1}\,dx = \int \frac{2}{7}+\frac{\frac{5}{7}}{7x+1}\,dx$$ (5)

$$\int \frac{2}{7}+\frac{5}{49}\left(\frac{7}{7x+1}\right)dx$$

$$\frac{2}{7}x+\frac{5}{49}\ln(|7x+1|)+C$$

$$\int \frac{x+1}{x^2+3x+2}\,dx = \int \frac{(x+1)}{(x+1)(x+2)}\,dx$$ (11)

$$\int \frac{1}{(x+2)}\,dx = \ln(|x+2|)+C$$