

Algebraic Geometry and Commutative Algebra.

Joseph Bernstein

Spring 2011

Course description:

This is a year long basic course in algebraic geometry for toar sheny complemented by necessary facts from commutative algebra.

Books. In my exposition I mostly follow the book:

Algebraic varieties by G.R.Kempf, Cambridge University Press (London Math. Society, Lecture Notes Series, v.172).

Sometimes for exercises I will use the book **Introduction to commutative algebra** by M.F. Atiyah and I.G.MacDonald.

Prerequisites. Good knowledge of linear algebra. Knowledge of basic notions of algebra - field, characteristic of a field, algebraically closed field, polynomial algebra. In the process of the course some geometric intuition will be useful; so the familiarity with basic notions of differential geometry will be helpful. Most of the necessary facts from commutative algebra will be proved in class.

Home assignments. I will be giving problem assignments weekly. These problem assignments are the integral part of the course - they will contain many important points for which there is not enough time in the course itself.

The grades for home assignments will be a factor in the final grade for the course.

Exams. There will be a midterm exam in class and a final take home exam.

Syllabus of Part I. (Spring semester 2011)

Affine algebraic varieties

Zariski topology

Noether's normalization lemma

Hilbert's basis theorem and Nullstellensatz

Projective varieties and general algebraic varieties

Products of algebraic varieties

Separated and complete varieties

Decomposition into irreducible components

Dimension - different definitions and properties

Principal ideal theorem

Smooth points and tangent spaces

Degree of a projective variety

Classical examples of algebraic varieties

Elements of Schemes theory

Syllabus of Part II. (Fall semester 2011):

Algebraic curves and their non-singular models

Riemann-Roch theorem - elementary approach
Sheaves
Coherent sheaves and localization. Serre's lemma
Cohomologies and elements of homological algebra.
Higher cohomological operations with sheaves. Base change
Different versions of Riemann-Roch theorem and its applications.
Jacobians of curves
Weil's proof of Riemann hypothesis for curves over finite fields.