Exercise VI

In the following exercises, write down the given numbers in the form $x + iy$.

Problems:

1. $(1+i)(1-i)$.
2. $(2+i)(3-4i)$.
3. $\left(\sqrt{2}-i\right) - i\left(1+\sqrt{2}i\right)$.
4. $\dfrac{1+i}{1-i}$.
5. $\dfrac{2+i}{3-4i}$.
6. $\dfrac{1+2i}{3-4i} + \dfrac{2-i}{5i}$.

Answers:

1. 2.
2. $10 - 5i$.
3. $-2i$.
4. i.
5. $\frac{2}{25} + \frac{11}{25}i$.
6. $-\frac{2}{5}$.

In the following exercises, write down the number $z = x + iy \in \mathbb{C}$ in the exponential form ($z = re^{i\varphi}$).

Problems:

1. $z = -i$.

2. $z = -2$.

3. $z = \sqrt{3} - i$.

4. $z = 1 - i\sqrt{3}$.

5. $z = 2 + 5i$.

6. $z = -2 + 5i$.

7. $z = 2 - 5i$.

8. $z = -2 - 5i$.

Answers:

1. $x = e^{-i\frac{\pi}{2}}$.

2. $x = 2e^{i\pi}$.

3. $x = 2e^{-i\frac{\pi}{6}}$.

4. $x = 2e^{-i\frac{\pi}{3}}$.

5. $x = \sqrt{29}e^{i \arctan \frac{5}{2}}$.

6. $x = \sqrt{29}e^{-i\left(\pi - \arctan \frac{5}{2}\right)}$.

7. $x = \sqrt{29}e^{-i \arctan \frac{5}{2}}$.

8. $x = \sqrt{29}e^{-i\left(\pi + \arctan \frac{5}{2}\right)}$.

Exercise VII

In the following exercises, draw the appropriate pictures on the plane of complex numbers:

Problems:

1. $|z| > 5$.
2. $|z - 2 + i| \leqslant 1$.
3. $|2z + 3| > 4$.
4. $\Im z > 1$.
5. $\Im z = 1$.
6. $|\arg z| < \frac{\pi}{4}$.
7. $|z + 1| = |2z - 1|$.
8. $|z - 4| \geqslant |z|$.

In the following exercises, write down the given expression in the Cartesian form $(x + iy)$:

Problems:

1. $\left(\frac{\sqrt{3}-i}{\sqrt{3}+i}\right)^4 \left(\frac{1+i}{1-i}\right)^5$.
2. $\left(\frac{1+i}{1-i}\right)^2 - 2\left(\frac{1-i}{1+i}\right)^3$.
3. $\frac{(2+i)(3-2i)(4-i)}{(1-i)^2}$.

Answers:

1. $\frac{\sqrt{3}}{2} - \frac{i}{2}$. 2. $-3-2i$ 3. $-\frac{15}{2}+5i$