1 Suppose that k is a field of characteristic $\neq 2$. Decompose into irreducible components the closed set $X \subset \mathbb{A}^3$ defined by $x^2 + y^2 + z^2 = 0$, $x^2 - y^2 - z^2 + 1 = 0$.

2 Prove that if X is the closed set of Exercise 4 of Section 2.4 then the elements of the field $k(X)$ can be expressed in a unique way in the form $u(x) + v(x)y$ where $u(x)$ and $v(x)$ are arbitrary rational functions of x.

3 Prove that the maps f of Exercises 3, 4 and 6 of Section 2.4 are birational.

4 Decompose into irreducible components the closed set $X \subset \mathbb{A}^3$ defined by $y^2 = xz$, $z^2 = y^3$. Prove that all its components are birational to $\mathbb{A}^1$.

5 Let $X \subset \mathbb{A}^n$ be the hypersurface defined by an equation $f_{n-1}(T_1, \ldots, T_n) + f_n(T_1, \ldots, T_n) = 0$, where f_{n-1} and f_n are homogeneous polynomials of degrees $n-1$ and n. (A hypersurface of this form is called a *monoid*.) Prove that if X is irreducible then it is birational to $\mathbb{A}^{n-1}$. (Compare the case of plane curves treated in Section 1.4.)

6 At what points of the circle given by $x^2 + y^2 = 1$ is the rational function $(1 - y)/x$ regular?

7 At which points of the curve X defined by $y^2 = x^2 + x^3$ is the rational function $t = y/x$ regular? Prove that $y/x \notin k[X]$.