1 Prove that the Segre variety $\varphi(\mathbb{P}^n \times \mathbb{P}^m) \subset \mathbb{P}^N$ (where $N = (n+1)(m+1) - 1$) is not contained in any linear subspace strictly smaller than the whole of $\mathbb{P}^N$.

2 Consider the two maps of varieties $\mathbb{P}^1 \times \mathbb{P}^1 \to \mathbb{P}^1$ given by $p_1(x, y) = x$ and $p_2(x, y) = y$. Prove that $p_1(X) = p_2(X) = \mathbb{P}^1$ for any closed irreducible subset $X \subset \mathbb{P}^1 \times \mathbb{P}^1$, unless X is of one of the following types: (a) a point $(x_0, y_0) \in \mathbb{P}^1 \times \mathbb{P}^1$; (b) a line $x_0 \times \mathbb{P}^1$ for $x_0 \in \mathbb{P}^1$ a fixed point; (c) a line $\mathbb{P}^1 \times y_0$.

3 Verify Theorem 1.10, Corollary 1.1 directly for the case $X = \mathbb{P}^n$.

4 Let $X = \mathbb{A}^2 \setminus x$ where x is a point. Prove that X is not isomorphic to an affine nor a projective variety (compare Exercise 3 of Section 4.5).

5 The same question as Exercise 4, for $X = \mathbb{P}^2 \setminus x$.

6 The same question as Exercise 4, for $X = \mathbb{P}^1 \times \mathbb{A}^1$.

7 Is the map $f\colon \mathbb{A}^1 \to X$ finite, where X is given by $y^2 = x^3$, and f by $f(t) = (t^2, t^3)$.

8 Let $X \subset \mathbb{A}^r$ be a hypersurface of $\mathbb{A}^r$ and L a line of $\mathbb{A}^r$ through the origin. Let φ_L be the map projecting X parallel to L to an $(r-1)$-dimensional subspace not containing L. Write S for the set of all lines L such that φ_L is not finite. Prove that S is an algebraic variety. [Hint: Prove that $S = \overline{X} \cap \mathbb{P}^{r-1}_\infty$.] Find S if $r = 2$ and X is given by $xy = 1$.

9 Prove that any intersection of affine open subsets is affine. [Hint: Use Example 1.20.]

10 Prove that forms of degree $m = kl$ in $n+1$ variables that are lth powers of forms correspond to the points of a closed subset of $\mathbb{P}^N$, where $N = \binom{n+m}{m} - 1 = \nu_{n,m}$.

11 Let $f\colon X \to Y$ be a regular map of affine varieties. Prove that the inverse image of a principal affine open set is a principal affine open set.