159. Find the sum of the cofactors of all elements of the determinants:

$$\text{(a)} \begin{vmatrix} a_1 & 0 & 0 & \dots & 0 \\ 0 & a_2 & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & a_n \end{vmatrix}; \quad \text{(b)} \begin{vmatrix} 0 & 0 & \dots & 0 & a_1 \\ 0 & 0 & \dots & a_2 & 0 \\ \dots & \dots & \dots & \dots & \dots \\ a_n & 0 & \dots & 0 & 0 \end{vmatrix}.$$

160. Expand the following determinant by the elements of the third row and evaluate:

$$\begin{vmatrix} 1 & 0 & -1 & -1 \\ 0 & -1 & -1 & 1 \\ a & b & c & d \\ -1 & -1 & 1 & 0 \end{vmatrix}.$$

161. Expand the determinant

$$\begin{vmatrix} 2 & 1 & 1 & x \\ 1 & 2 & 1 & y \\ 1 & 1 & 2 & z \\ 1 & 1 & 1 & t \end{vmatrix}$$

by the elements of the last column and evaluate.

162. Expand the determinant

$$\begin{vmatrix} a & 1 & 1 & 1 \\ b & 0 & 1 & 1 \\ c & 1 & 0 & 1 \\ d & 1 & 1 & 0 \end{vmatrix}$$

by the elements of the first column and evaluate.

Sec. 5. Computing Determinants

Compute the determinants:

***163.** $\begin{vmatrix} 13547 & 13647 \\ 28423 & 28523 \end{vmatrix}$. **164.** $\begin{vmatrix} 246 & 427 & 327 \\ 1014 & 543 & 443 \\ -342 & 721 & 621 \end{vmatrix}$.

165. $\begin{vmatrix} 3 & 1 & 1 & 1 \\ 1 & 3 & 1 & 1 \\ 1 & 1 & 3 & 1 \\ 1 & 1 & 1 & 3 \end{vmatrix}$. **166.** $\begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \\ 1 & 3 & 6 & 10 \\ 1 & 4 & 10 & 20 \end{vmatrix}$. **167.** $\begin{vmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \\ 3 & 4 & 1 & 2 \\ 4 & 1 & 2 & 3 \end{vmatrix}$

168. $\begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \\ 1 & 4 & 9 & 16 \\ 1 & 8 & 27 & 64 \end{vmatrix}$. **169.** $\begin{vmatrix} 1 & 2 & 3 & 4 \\ -2 & 1 & -4 & 3 \\ 3 & -4 & -1 & 2 \\ 4 & 3 & -2 & -1 \end{vmatrix}$.

170. $\begin{vmatrix} 2 & 1 & 1 & 1 & 1 \\ 1 & 3 & 1 & 1 & 1 \\ 1 & 1 & 4 & 1 & 1 \\ 1 & 1 & 1 & 5 & 1 \\ 1 & 1 & 1 & 1 & 6 \end{vmatrix}$. **171.** $\begin{vmatrix} 5 & 6 & 0 & 0 & 0 \\ 1 & 5 & 6 & 0 & 0 \\ 0 & 1 & 5 & 6 & 0 \\ 0 & 0 & 1 & 5 & 6 \\ 0 & 0 & 0 & 1 & 5 \end{vmatrix}$.

172. $\begin{vmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & a & b \\ 1 & a & 0 & c \\ 1 & b & c & 0 \end{vmatrix}$. **173.** $\begin{vmatrix} x & y & x+y \\ y & x+y & x \\ x+y & x & y \end{vmatrix}$.

174. $\begin{vmatrix} x & 0 & -1 & 1 & 0 \\ 1 & x & -1 & 1 & 0 \\ 1 & 0 & x-1 & 0 & 1 \\ 0 & 1 & -1 & x & 1 \\ 0 & 1 & -1 & 0 & x \end{vmatrix}$. **175.** $\begin{vmatrix} 1+x & 1 & 1 & 1 \\ 1 & 1-x & 1 & 1 \\ 1 & 1 & 1+z & 1 \\ 1 & 1 & 1 & 1-z \end{vmatrix}$.

176. $\begin{vmatrix} 1 & 1 & 2 & 3 \\ 1 & 2-x^2 & 2 & 3 \\ 2 & 3 & 1 & 5 \\ 2 & 3 & 1 & 9-x^2 \end{vmatrix}$.

177. $\begin{vmatrix} \cos(a-b) & \cos(b-c) & \cos(c-a) \\ \cos(a+b) & \cos(b+c) & \cos(c+a) \\ \sin(a+b) & \sin(b+c) & \sin(c+a) \end{vmatrix}$.

178.
$$\begin{vmatrix} 0 & a & b & c \\ -a & 0 & d & e \\ -b & -d & 0 & f \\ -c & -e & -f & 0 \end{vmatrix}.$$

***179.**
$$\begin{vmatrix} 1 & 2 & 3 & \dots & n \\ -1 & 0 & 3 & \dots & n \\ -1 & -2 & 0 & \dots & n \\ \dots & \dots & \dots & \dots & \dots \\ -1 & -2 & -3 & \dots & 0 \end{vmatrix}.$$

***180.**
$$\begin{vmatrix} 1 & a_1 & a_2 & & a_n \\ 1 & a_1+b_1 & a_2 & & a_n \\ 1 & a_1 & a_2+b_2 & \dots & a_n \\ \dots & \dots & \dots & \dots & \dots \\ 1 & a_1 & a_2 & \dots & a_n+b_n \end{vmatrix}.$$

***181.**
$$\begin{vmatrix} 1 & x_1 & x_2 & \dots & x_{n-1} & x_n \\ 1 & x & x_2 & \dots & x_{n-1} & x_n \\ 1 & x_1 & x & \dots & x_{n-1} & x_n \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 1 & x_1 & x_2 & \dots & x & x_n \\ 1 & x_1 & x_2 & \dots & x_{n-1} & x \end{vmatrix}.$$

***182.**
$$\begin{vmatrix} 1 & 2 & 3 & \dots & n-1 & n \\ 1 & 3 & 3 & \dots & n-1 & n \\ 1 & 2 & 5 & \dots & n-1 & n \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 1 & 2 & 3 & \dots & 2n-3 & n \\ 1 & 2 & 3 & \dots & n-1 & 2n-1 \end{vmatrix}.$$

***183.**
$$\begin{vmatrix} 1 & 2 & 2 & \dots & 2 \\ 2 & 2 & 2 & \dots & 2 \\ 2 & 2 & 3 & \dots & 2 \\ \dots & \dots & \dots & \dots & \dots \\ 2 & 2 & 2 & \dots & n \end{vmatrix}.$$

***184.**
$$\begin{vmatrix} 1 & b_1 & 0 & 0 & \dots & 0 & 0 \\ -1 & 1-b_1 & b_2 & 0 & \dots & 0 & 0 \\ 0 & -1 & 1-b_2 & b_3 & \dots & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & \dots & 1-b_{n-1} & b_n \\ 0 & 0 & 0 & 0 & \dots & -1 & 1-b_n \end{vmatrix}.$$

***185.**
$$\begin{vmatrix} a & a+h & a+2h & \dots & a+(n-1)h \\ -a & a & 0 & \dots & 0 \\ 0 & -a & a & \dots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \dots & a \end{vmatrix}.$$

***186.**
$$\begin{vmatrix} a & -(a+h) & \dots & (-1)^{n-1}[a+(n-1)h] \\ a & a & \dots & 0 \\ 0 & a & \dots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \dots & a \end{vmatrix}.$$

***187.**
$$\begin{vmatrix} 1 & C_n^1 & C_n^2 & C_n^3 & \dots & C_a^{n-2} & C_n^{n-1} & C_n^n \\ 1 & C_{n-1}^1 & C_{n-1}^2 & C_{n-1}^3 & \dots & C_{n-1}^{n-2} & C_{n-1}^{n-1} & 0 \\ 1 & C_{n-2}^1 & C_{n-2}^2 & C_{n-2}^3 & \dots & C_{n-2}^{n-2} & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 1 & C_2^1 & C_2^2 & 0 & \dots & 0 & 0 & 0 \\ 1 & C_1^1 & 0 & 0 & \dots & 0 & 0 & 0 \\ a_0 & a_1 & a_2 & a_3 & \dots & a_{n-2} & a_{n-1} & a_n \end{vmatrix}.$$

***188.**
$$\begin{vmatrix} a_0 & -1 & 0 & \dots & 0 & 0 \\ a_1 & x & -1 & \dots & 0 & 0 \\ a_2 & 0 & x & \dots & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{n-1} & 0 & 0 & \dots & x & -1 \\ a_n & 0 & 0 & \dots & 0 & x \end{vmatrix}.$$

***189.**
$$\begin{vmatrix} n & n-1 & n-2 & \dots & 3 & 2 & 1 \\ -1 & x & 0 & \dots & 0 & 0 & 0 \\ 0 & -1 & x & \dots & 0 & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \dots & -1 & x & 0 \\ 0 & 0 & 0 & \dots & 0 & -1 & x \end{vmatrix}.$$

***190.** Compute the difference $f(x+1)-f(x)$, where

$$f(x)=\begin{vmatrix} 1 & 0 & 0 & 0 & \dots & 0 & x \\ 1 & 2 & 0 & 0 & \dots & 0 & x^2 \\ 1 & 3 & 3 & 0 & \dots & 0 & x^3 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 1 & n & C_n^2 & C_n^3 & \dots & C_n^{n-1} & x^n \\ 1 & n+1 & C_{n+1}^2 & C_{n+1}^3 & \dots & C_{n+1}^{n-1} & x^{n+1} \end{vmatrix}.$$

Compute the determinants:

***191.** $$\begin{vmatrix} x & a_1 & a_2 & \dots & a_{n-1} & 1 \\ a_1 & x & a_2 & \dots & a_{n-1} & 1 \\ a_1 & a_2 & x & \dots & a_{n-1} & 1 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ a_1 & a_2 & a_3 & \dots & x & 1 \\ a_1 & a_2 & a_3 & \dots & a_n & 1 \end{vmatrix}.$$

***192.** $$\begin{vmatrix} x & a & a & \dots & a \\ a & x & a & \dots & a \\ a & a & x & \dots & a \\ \dots & \dots & \dots & \dots & \dots \\ a & a & a & \dots & x \end{vmatrix}.$$

193. $$\begin{vmatrix} x & a & a & \dots & a & a \\ -a & x & a & \dots & a & a \\ -a & -a & x & \dots & a & a \\ \dots & \dots & \dots & \dots & \dots & \dots \\ -a & -a & -a & \dots & -a & x \end{vmatrix}.$$

***194.** $$\begin{vmatrix} -a_1 & a_1 & 0 & \dots & 0 & 0 \\ 0 & -a_2 & a_2 & \dots & 0 & 0 \\ 0 & 0 & -a_3 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & -a_n & a_n \\ 1 & 1 & 1 & \dots & 1 & 1 \end{vmatrix}.$$

***195.** $$\begin{vmatrix} a_1 & -a_2 & 0 & \dots & 0 & 0 \\ 0 & a_2 & -a_3 & \dots & 0 & 0 \\ 0 & 0 & a_3 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & a_{n-1} & -a_n \\ 1 & 1 & 1 & \dots & 1 & 1+a_n \end{vmatrix}.$$

***196.** $$\begin{vmatrix} h & -1 & 0 & 0 & \dots & 0 \\ hx & h & -1 & 0 & \dots & 0 \\ hx^2 & hx & h & -1 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ hx^n & hx^{n-1} & hx^{n-2} & hx^{n-3} & \dots & h \end{vmatrix}.$$

***197.** $$\begin{vmatrix} 0 & 1 & 1 & \dots & 1 & 1 \\ 1 & 0 & x & \dots & x & x \\ 1 & x & 0 & \dots & x & x \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 1 & x & x & \dots & 0 & x \\ 1 & x & x & \dots & x & 0 \end{vmatrix}.$$

***198.** $$\begin{vmatrix} 0 & 1 & 1 & \dots & 1 \\ 1 & 0 & a_1+a_2 & \dots & a_1+a_n \\ 1 & a_2+a_1 & 0 & \dots & a_2+a_n \\ \dots & \dots & \dots & \dots & \dots \\ 1 & a_n+a_1 & a_n+a_2 & \dots & 0 \end{vmatrix}.$$

***199.** $$\begin{vmatrix} 1 & 2 & 3 & \dots & n-1 & n \\ 1 & 1 & 1 & \dots & 1 & 1-n \\ 1 & 1 & 1 & \dots & 1-n & 1 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 1 & 1-n & 1 & \dots & 1 & 1 \end{vmatrix}.$$

***200.** $$\begin{vmatrix} 2 & 1-\frac{1}{n} & 1-\frac{1}{n} & \dots & 1-\frac{1}{n} \\ 1-\frac{1}{n} & 2 & 1-\frac{1}{n} & \dots & 1-\frac{1}{n} \\ \dots & \dots & \dots & \dots & \dots \\ 1-\frac{1}{n} & 1-\frac{1}{n} & 1-\frac{1}{n} & \dots & 2 \end{vmatrix}$$

(order $n+1$).

***201.** $$\begin{vmatrix} 1 & a & a^2 & a^3 & \dots & a^n \\ x_{11} & 1 & a & a^2 & \dots & a^{n-1} \\ x_{21} & x_{22} & 1 & a & \dots & a^{n-2} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ x_{n1} & x_{n2} & x_{n3} & x_{n4} & \dots & 1 \end{vmatrix}.$$

***202.**
$$\begin{vmatrix} 1 & 2 & 3 & 4 & \dots & n \\ 2 & 1 & 2 & 3 & \dots & n-1 \\ 3 & 2 & 1 & 2 & \dots & n-2 \\ 4 & 3 & 2 & 1 & \dots & n-3 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ n & n-1 & n-2 & n-3 & \dots & 1 \end{vmatrix}.$$

***203.**
$$\begin{vmatrix} a_0 & b_1 & 0 & 0 & \dots & 0 & 0 \\ a_1 & -b_0 & b_2 & 0 & \dots & 0 & 0 \\ a_2 & 0 & -b_1 & b_3 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ a_{n-1} & 0 & 0 & 0 & \dots & -b_{n-2} & b_n \\ a_n & 0 & 0 & 0 & \dots & 0 & -b_{n-1} \end{vmatrix}.$$

***204.**
$$\begin{vmatrix} a & a^2 & 0 & 0 & \dots & 0 & 0 \\ 1 & 2a+b & (a+b)^2 & 0 & \dots & 0 & 0 \\ 0 & 1 & 2a+3b & (a+2b)^2 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & 0 & \dots & 2a+(2n-1)b & (a+nb)^2 \\ 0 & 0 & 0 & 0 & \dots & 1 & 2a+(2n+1)b \end{vmatrix}.$$

***205.**
$$\begin{vmatrix} x & y & 0 & \dots & 0 & 0 \\ 0 & x & y & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & x & y \\ y & 0 & 0 & \dots & 0 & x \end{vmatrix}.$$

***206.**
$$\begin{vmatrix} 1+x_1y_1 & 1+x_1y_2 & \dots & 1+x_1y_n \\ 1+x_2y_1 & 1+x_2y_2 & \dots & 1+x_2y_n \\ \dots & \dots & \dots & \dots \\ 1+x_ny_1 & 1+x_ny_2 & \dots & 1+x_ny_n \end{vmatrix}.$$

207.
$$\begin{vmatrix} a_1-b_1 & a_1-b_2 & \dots & a_1-b_n \\ a_2-b_1 & a_2-b_2 & \dots & a_2-b_n \\ \dots & \dots & \dots & \dots \\ a_n-b_1 & a_n-b_2 & \dots & a_n-b_n \end{vmatrix}.$$

***208.**
$$\begin{vmatrix} 1+a_1+x_1 & a_1+x_2 & \dots & a_2+x_n \\ a_2+x_1 & 1+a_2+x_2 & \dots & a_2+x_n \\ \dots & \dots & \dots & \dots \\ a_n+x_1 & a_n+x_2 & \dots & 1+a_n+x_n \end{vmatrix}.$$

209.
$$\begin{vmatrix} a^n-\alpha & a^{n+1}-\alpha & \dots & a^{n+p-1}-\alpha \\ a^{n+p}-\alpha & a^{n+p+1}-\alpha & \dots & a^{n+2p-1}-\alpha \\ \dots & \dots & \dots & \dots \\ a^{n+p(p-1)}-\alpha & a^{n+p(p-1)+1}-\alpha & \dots & a^{n+p^2-1}-\alpha \end{vmatrix}.$$

210. Prove that the determinant

$$\begin{vmatrix} f_1(a_1) & f_1(a_2) & \dots & f_1(a_n) \\ f_2(a_1) & f_2(a_2) & \dots & f_2(a_n) \\ \dots & \dots & \dots & \dots \\ f_n(a_1) & f_n(a_2) & \dots & f_n(a_n) \end{vmatrix}$$

is equal to zero if $f_1(x), f_2(x), \dots, f_n(x)$ are polynomials in x, each of degree not exceeding $n-2$, and the numbers $a_1, a_2, \dots, a_n$ are arbitrary.

Compute the determinants:

***211.**
$$\begin{vmatrix} 1 & 2 & 3 & 4 & \dots & n-1 & n \\ -1 & x & 0 & 0 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & 0 & \dots & x & 0 \\ 0 & 0 & 0 & 0 & \dots & -1 & x \end{vmatrix}.$$

***212.**
$$\begin{vmatrix} a_1+x_1 & a_2 & a_3 & \dots & a_{n-1} & a_n \\ -x_1 & x_2 & 0 & \dots & 0 & 0 \\ 0 & -x_2 & x_3 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & -x_{n-1} & x_n \end{vmatrix}.$$

***213.**
$$\begin{vmatrix} a_0 & a_1 & a_2 & \dots & a_{n-1} & a_n \\ -y_1 & x_1 & 0 & \dots & 0 & 0 \\ 0 & -y_2 & x_2 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & -y_n & x_n \end{vmatrix}.$$

***214.**
$$\begin{vmatrix} 0 & 1 & 1 & \dots & 1 \\ 1 & a_1 & 0 & \dots & 0 \\ 1 & 0 & a_2 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 1 & 0 & 0 & \dots & a_n \end{vmatrix}.$$

***215.**
$$\begin{vmatrix} n!\,a_0 & (n-1)!\,a_1 & (n-2)!\,a_2 & \dots & a_n \\ -n & x & 0 & \dots & 0 \\ 0 & -(n-1) & x & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & x \end{vmatrix}.$$

216. $\begin{vmatrix} 1 & 0 & 0 & 0 & 1 \\ 1 & a_1 & 0 & 0 & 0 \\ 1 & 1 & a_2 & 0 & 0 \\ 1 & 0 & 1 & a_3 & 0 \\ 1 & 0 & 0 & 1 & a_4 \end{vmatrix}$. Write an nth-order determinant of this structure and compute it.

Compute the determinants:

217.

$$\begin{vmatrix} \alpha+\beta & \alpha\beta & 0 & \dots & 0 & 0 \\ 1 & \alpha+\beta & \alpha\beta & \dots & 0 & 0 \\ 0 & 1 & \alpha+\beta & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & 1 & \alpha+\beta \end{vmatrix}.$$

218.

$$\begin{vmatrix} 2 & 1 & 0 & 0 & \dots & 0 \\ 1 & 2 & 1 & 0 & \dots & 0 \\ 0 & 1 & 2 & 1 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & 0 & \dots & 2 \end{vmatrix}.$$

***219.** $$\begin{vmatrix} 2\cos\theta & 1 & 0 & \dots & 0 & 0 \\ 1 & 2\cos\theta & 1 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & 1 & 2\cos\theta \end{vmatrix}.$$

220. $$\begin{vmatrix} \cos\theta & 1 & 0 & \dots & 0 \\ 1 & 2\cos\theta & 1 & \dots & 0 \\ 0 & 1 & 2\cos\theta & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & 2\cos\theta \end{vmatrix}.$$

***221.**

$$\begin{vmatrix} x & 1 & 0 & \dots & 0 \\ 1 & x & 1 & \dots & 0 \\ 0 & 1 & x & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & x \end{vmatrix}.$$

***222.**

$$\begin{vmatrix} x_1y_1 & x_1y_2 & x_1y_3 & \dots & x_1y_n \\ x_1y_2 & x_2y_2 & x_2y_3 & \dots & x_2y_n \\ x_1y_3 & x_2y_3 & x_3y_3 & \dots & x_3y_n \\ \dots & \dots & \dots & \dots & \dots \\ x_1y_n & x_2y_n & x_3y_n & \dots & x_ny_n \end{vmatrix}.$$

***223.** $$\begin{vmatrix} 1+a_1 & 1 & 1 & \dots & 1 \\ 1 & 1+a_2 & 1 & \dots & 1 \\ 1 & 1 & 1+a_3 & \dots & 1 \\ \dots & \dots & \dots & \dots & \dots \\ 1 & 1 & 1 & \dots & 1+a_n \end{vmatrix}.$$

224. $$\begin{vmatrix} 1 & 1 & & 1 & a_1+1 \\ 1 & 1 & \dots & a_2+1 & 1 \\ \dots & \dots & \dots & \dots & \dots \\ 1 & a_{n-1}+1 & \dots & 1 & 1 \\ a_n+1 & 1 & \dots & 1 & 1 \end{vmatrix}.$$

***225.** $$\begin{vmatrix} a_1 & x & x & \dots & x \\ x & a_2 & x & \dots & x \\ x & x & a_3 & \dots & x \\ \dots & \dots & \dots & \dots & \dots \\ x & x & x & \dots & a_n \end{vmatrix}.$$

***226.** $$\begin{vmatrix} x_1 & a_2 & a_3 & \dots & a_{n-1} & a_n \\ a_1 & x_2 & a_3 & \dots & a_{n-1} & a_n \\ a_1 & a_2 & x_3 & \dots & a_{n-1} & a_n \\ \dots & \dots & \dots & \dots & \dots & \dots \\ a_1 & a_2 & a_3 & \dots & x_{n-1} & a_n \\ a_1 & a_2 & a_3 & \dots & a_{n-1} & x_n \end{vmatrix}.$$

***227.** $$\begin{vmatrix} x_1 & a_2 b_1 & a_3 b_1 & \dots & a_n b_1 \\ a_1 b_2 & x_2 & a_3 b_2 & \dots & a_n b_2 \\ a_1 b_3 & a_2 b_3 & x_3 & \dots & a_n b_3 \\ \dots & \dots & \dots & \dots & \dots \\ a_1 b_n & a_2 b_n & a_3 b_n & \dots & x_n \end{vmatrix}.$$

***228.** $$\begin{vmatrix} x_1-m & x_2 & x_3 & \dots & x_n \\ x_1 & x_2-m & x_3 & \dots & x_n \\ x_1 & x_2 & x_3-m & \dots & x_n \\ \dots & \dots & \dots & \dots & \dots \\ x_1 & x_2 & x_3 & \dots & x_n-m \end{vmatrix}.$$

229. Solve the equation

$$\begin{vmatrix} a_1 & a_2 & \dots & a_{n-1} & a_n-\alpha_n x \\ a_1 & a_2 & \dots & a_{n-1}-\alpha_{n-1} x & a_n \\ \dots & \dots & \dots & \dots & \dots \\ a_1-\alpha_1 x & a_2 & \dots & a_{n-1} & a_n \end{vmatrix} = 0.$$

Compute the determinants:

***230.** $$\begin{vmatrix} a & 0 & 0 & \dots & 0 & 0 & b \\ 0 & a & 0 & \dots & 0 & b & 0 \\ 0 & 0 & a & \dots & b & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & b & \dots & a & 0 & 0 \\ 0 & b & 0 & \dots & 0 & a & 0 \\ b & 0 & 0 & \dots & 0 & 0 & a \end{vmatrix}$$ (of order $2n$).

***231.**
$$\begin{vmatrix} 1 & -b & -b & -b & & -b \\ 1 & na & -2b & -3b & \dots & -(n-1)b \\ 1 & (n-1)a & a & -3b & \dots & -(n-1)b \\ 1 & (n-2)a & a & a & \dots & -(n-1)b \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 1 & 2a & a & a & \dots & a \end{vmatrix}.$$

***232.**
$$\begin{vmatrix} (x-a_1)^2 & a_2^2 & & a_n^2 \\ a_1^2 & (x-a_2)^2 & \dots & a_n^2 \\ \dots & \dots & \dots & \dots \\ a_1^2 & a_2^2 & \dots & (x-a_n)^2 \end{vmatrix}.$$

***233.**
$$\begin{vmatrix} (x-a_1)^2 & a_1 a_2 & \dots & a_1 a_n \\ a_1 a_2 & (x-a_2)^2 & \dots & a_2 a_n \\ \dots & \dots & \dots & \dots \\ a_1 a_n & a_2 a_n & \dots & (x-a_n)^2 \end{vmatrix}.$$

***234.**
$$\begin{vmatrix} 1-b_1 & b_2 & 0 & 0 & \dots & 0 \\ -1 & 1-b_2 & b_3 & 0 & \dots & 0 \\ 0 & -1 & 1-b_3 & b_4 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & 0 & \dots & 1-b_n \end{vmatrix}.$$

***235.**
$$\begin{vmatrix} 0 & a_2 & a_3 & a_4 & \dots & a_{n-1} & a_n \\ b_1 & 0 & a_3 & a_4 & \dots & a_{n-1} & a_n \\ b_1 & b_2 & 0 & a_4 & \dots & a_{n-1} & a_n \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ b_1 & b_2 & b_3 & b_4 & \dots & 0 & a_n \\ b_1 & b_2 & b_3 & b_4 & \dots & b_{n-1} & 0 \end{vmatrix}.$$

***236.**
$$\begin{vmatrix} 1 & 2 & 3 & 4 & 5 & \dots & n \\ 1 & 1 & 2 & 3 & 4 & \dots & n-1 \\ 1 & x & 1 & 2 & 3 & \dots & n-2 \\ 1 & x & x & 1 & 2 & \dots & n-3 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 1 & x & x & x & x & \dots & 1 \end{vmatrix}.$$

***237.**
$$\begin{vmatrix} 1 & 2 & 3 & 4 & \dots & n \\ x & 1 & 2 & 3 & \dots & n-1 \\ x & x & 1 & 2 & \dots & n-2 \\ x & x & x & 1 & \dots & n-3 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ x & x & x & x & \dots & 1 \end{vmatrix}.$$

***238.** $$\begin{vmatrix} a_0 x^n & a_1 x^{n-1} & a_2 x^{n-2} & \dots & a_{n-1} x & a_n \\ a_0 x & b_1 & 0 & & 0 & 0 \\ a_0 x^2 & a_1 x & b_2 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ a_0 x^{n-1} & a_1 x^{n-2} & a_2 x^{n-3} & \dots & b_{n-1} & 0 \\ a_0 x^n & a_1 x^{n-1} & a_2 x^{n-2} & \dots & a_{n-1} x & b_n \end{vmatrix}.$$

***239.** Prove that the determinant

$$\begin{vmatrix} a_{00} x^n & a_{01} x^{n-1} & a_{02} x^{n-2} & \dots & a_{0n} \\ a_{10} x & a_{11} & 0 & & 0 \\ a_{20} x^2 & a_{21} x & a_{22} & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ a_{n0} x^n & a_{n1} x^{n-1} & a_{n2} x^{n-2} & \dots & a_{nn} \end{vmatrix} = x^n \cdot \begin{vmatrix} a_{00} & a_{01} & a_{02} & \dots & a_{0n} \\ a_{10} & a_{11} & 0 & \dots & 0 \\ a_{20} & a_{21} & a_{22} & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ a_{n0} & a_{n1} & a_{n2} & \dots & a_{nn} \end{vmatrix}.$$

Compute the determinants:

***240.** $$\begin{vmatrix} 1 & 1 & 1 & \dots & 1 \\ 1 & C_2^1 & C_3^1 & \dots & C_n^1 \\ 1 & C_3^2 & C_4^2 & \dots & C_{n+1}^2 \\ \dots & \dots & \dots & \dots & \dots \\ 1 & C_n^{n-1} & C_{n+1}^{n-1} & \dots & C_{2n-2}^{n-1} \end{vmatrix}.$$

***241.** $$\begin{vmatrix} 1 & 1 & \dots & 1 \\ C_m^1 & C_{m+1}^1 & \dots & C_{m+n}^1 \\ C_{m+1}^2 & C_{m+2}^2 & \dots & C_{m+n+1}^2 \\ \dots & \dots & \dots & \dots \\ C_{m+n-1}^n & C_{m+n}^n & \dots & C_{m+2n-1}^n \end{vmatrix}.$$

***242.** $$\begin{vmatrix} 1 & 1 & 0 & 0 & \dots & 0 \\ 1 & C_2^1 & C_2^2 & 0 & \dots & 0 \\ 1 & C_3^1 & C_3^2 & C_3^3 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 1 & C_n^1 & C_n^2 & C_n^3 & \dots & C_n^{n-1} \end{vmatrix}.$$

***243.** $$\begin{vmatrix} C_m^k & C_m^{k+1} & \dots & C_m^{k+n} \\ C_{m+1}^k & C_{m+1}^{k+1} & \dots & C_{m+1}^{k+n} \\ \dots & \dots & \dots & \dots \\ C_{m+n}^k & C_{m+n}^{k+1} & \dots & C_{m+n}^{k+n} \end{vmatrix}.$$

***244.** $$\begin{vmatrix} C_{k+m}^m & C_{k+m+1}^m & \dots & C_{k+2m}^m \\ C_{k+m+1}^m & C_{k+m+2}^m & \dots & C_{k+2m+1}^m \\ \dots & \dots & \dots & \dots \\ C_{k+2m}^m & C_{k+2m+1}^m & \dots & C_{k+3m}^m \end{vmatrix}.$$

***245.** $$\begin{vmatrix} 1 & 0 & 0 & \dots & 0 & 1 \\ 1 & C_1^1 & 0 & \dots & 0 & x \\ 1 & C_2^1 & C_2^2 & \dots & 0 & x^2 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 1 & C_n^1 & C_n^2 & \dots & C_n^{n-1} & x^n \end{vmatrix}.$$

***246.**
$$\begin{vmatrix} 1 & 0 & 0 & 0 & \dots & 1 \\ 1 & 1! & 0 & 0 & \dots & x \\ 1 & 2 & 2! & 0 & \dots & x^2 \\ 1 & 3 & 3\cdot 2 & 3! & \dots & x^3 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 1 & n & n(n-1) & n(n-1)(n-2) & \dots & x^n \end{vmatrix}.$$

***247.**
$$\begin{vmatrix} \alpha & \alpha+\delta & \alpha+2\delta & \alpha+3\delta & \dots & \alpha+(n-1)\delta \\ \alpha & 2\alpha+\delta & 3\alpha+3\delta & 4\alpha+6\delta & \dots & C_n^1\alpha+C_n^2\delta \\ \alpha & 3\alpha+\delta & 6\alpha+4\delta & 10\alpha+10\delta & \dots & C_{n+1}^2\alpha+C_{n+1}^3\delta \\ \dots & \dots & \dots & \dots & \dots & \dots \\ \alpha & C_n^{n-1}\alpha+\delta & C_{n+1}^{n-1}\alpha+C_{n+1}^n\delta & C_{n+2}^{n-1}\alpha+C_{n+2}^n\delta & \dots & C_{2n-2}^{n-1}\alpha+C_{2n-2}^n\delta \end{vmatrix}.$$

***248.**
$$\begin{vmatrix} x & y & y & \dots & y & y \\ z & x & y & \dots & y & y \\ z & z & x & \dots & y & y \\ \dots & \dots & \dots & \dots & \dots & \dots \\ z & z & z & \dots & x & y \\ z & z & z & \dots & z & x \end{vmatrix}.$$

***249.**
$$\begin{vmatrix} a & a & a & \dots & a & 0 \\ a & a & a & \dots & 0 & b \\ \dots & \dots & \dots & \dots & \dots & \dots \\ a & 0 & b & \dots & b & b \\ 0 & b & b & \dots & b & b \end{vmatrix}.$$

250.
$$\begin{vmatrix} a_1 & x & x & \dots & x \\ y & a_2 & x & \dots & x \\ \dots & \dots & \dots & \dots & \dots \\ y & y & y & \dots & a_n \end{vmatrix}.$$

251.
$$\begin{vmatrix} c_1 & a & a & \dots & a & 1 \\ b & c_2 & a & \dots & a & 1 \\ b & b & c_3 & \dots & a & 1 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ b & b & b & \dots & c_n & 1 \\ 1 & 1 & 1 & \dots & 1 & 0 \end{vmatrix}.$$

***252.**
$$\begin{vmatrix} \lambda & a & a & a & \dots & a \\ b & \alpha & \beta & \beta & \dots & \beta \\ b & \beta & \alpha & \beta & \dots & \beta \\ b & \beta & \beta & \alpha & \dots & \beta \\ \dots & \dots & \dots & \dots & \dots & \dots \\ b & \beta & \beta & \beta & \dots & \alpha \end{vmatrix}.$$

***253.**
$$\begin{vmatrix} 1 & 2 & 3 & \dots & n \\ 2 & 3 & 4 & \dots & 1 \\ 3 & 4 & 5 & \dots & 2 \\ \dots & \dots & \dots & \dots & \dots \\ n & 1 & 2 & \dots & n-1 \end{vmatrix}.$$

***254.** $$\begin{vmatrix} a & a+h & a+2h & \dots & a+(n-1)h \\ a+h & a+2h & a+3h & \dots & a \\ a+2h & a+3h & a+4h & \dots & a+h \\ \dots & \dots & \dots & \dots & \dots \\ a+(n-1)h & a & a+h & \dots & a+(n-2)h \end{vmatrix}.$$

255. $$\begin{vmatrix} 1 & x & x^2 & \dots & x^{n-1} \\ x^{n-1} & 1 & x & \dots & x^{n-2} \\ \dots & \dots & \dots & \dots & \dots \\ x & x^2 & x^3 & \dots & 1 \end{vmatrix}.$$

***256.** $$\begin{vmatrix} a & b & c & d \\ b & a & d & c \\ c & d & a & b \\ d & c & b & a \end{vmatrix}.$$

257.

$$\begin{vmatrix} a & b & c & d & e & f & g & h \\ b & a & d & c & f & e & h & g \\ c & d & a & b & g & h & e & f \\ d & c & b & a & h & g & f & e \\ e & f & g & h & a & b & c & d \\ f & e & h & g & b & a & d & c \\ g & h & e & f & c & d & a & b \\ h & g & f & e & d & c & b & a \end{vmatrix}.$$

***258.**

$$\begin{vmatrix} x & a_1 & a_2 & \dots & a_n \\ a_1 & x & a_2 & \dots & a_n \\ \dots & \dots & \dots & \dots & \dots \\ a_1 & a_2 & a_3 & \dots & x \end{vmatrix}.$$

***259.** $$\begin{vmatrix} \cos^{n-1}\varphi_1 & \cos^{n-2}\varphi_1 & \dots & \cos\varphi_1 & 1 \\ \cos^{n-1}\varphi_2 & \cos^{n-2}\varphi_2 & \dots & \cos\varphi_2 & 1 \\ \dots & \dots & \dots & \dots & \dots \\ \cos^{n-1}\varphi_n & \cos^{n-2}\varphi_n & \dots & \cos\varphi_n & 1 \end{vmatrix}.$$

260. $$\begin{vmatrix} 1 & 1 & \dots & 1 \\ \sin\varphi_1 & \sin\varphi_2 & \dots & \sin\varphi_n \\ \sin^2\varphi_1 & \sin^2\varphi_2 & \dots & \sin^2\varphi_n \\ \dots & \dots & \dots & \dots \\ \sin^{n-1}\varphi_1 & \sin^{n-1}\varphi_2 & \dots & \sin^{n-1}\varphi_n \end{vmatrix}.$$

261. $$\begin{vmatrix} a^n & (a-1)^n & \dots & (a-n)^n \\ a^{n-1} & (a-1)^{n-1} & \dots & (a-n)^{n-1} \\ \dots & \dots & \dots & \dots \\ a & a-1 & \dots & a-n \\ 1 & 1 & \dots & 1 \end{vmatrix}.$$

262. $$\begin{vmatrix} (a_1+x)^n & (a_1+x)^{n-1} & & a_1+x & 1 \\ (a_2+x)^n & (a_2+x)^{n-1} & \dots & a_2+x & 1 \\ \dots & \dots & \dots & \dots & \dots \\ (a_{n+1}+x)^n & (a_{n+1}+x)^{n-1} & \dots & a_{n+1}+x & 1 \end{vmatrix}.$$

263. $$\begin{vmatrix} (2n-1)^n & (2n-2)^n & \dots & n^n & (2n)^n \\ (2n-1)^{n-1} & (2n-2)^{n-1} & \dots & n^{n-1} & (2n)^{n-1} \\ \dots & \dots & \dots & \dots & \dots \\ 2n-1 & 2n-2 & \dots & n & 2n \\ 1 & 1 & \dots & 1 & 1 \end{vmatrix}.$$

***264.** $$\begin{vmatrix} w_1 & a_1 & a_1^2 & \dots & a_1^{n-1} \\ w_2 & a_2 & a_2^2 & \dots & a_2^{n-1} \\ \dots & \dots & \dots & \dots & \dots \\ w_n & a_n & a_n^2 & \dots & a_n^{n-1} \end{vmatrix}.$$

***265.**

$$\begin{vmatrix} 1 & 1 & 1 & \dots & 1 \\ x_1+1 & x_2+1 & x_3+1 & \dots & x_n+1 \\ x_1^2+x_1 & x_2^2+x_2 & x_3^2+x_3 & \dots & x_n^2+x_n \\ \dots & \dots & \dots & \dots & \dots \\ x_1^{n-1}+x_1^{n-2} & x_2^{n-1}+x_2^{n-2} & x_3^{n-1}+x_3^{n-2} & \dots & x_n^{n-1}+x_n^{n-2} \end{vmatrix}.$$

266.

$$\begin{vmatrix} 1 & 1 & \dots & 1 \\ 1+\sin\varphi_1 & 1+\sin\varphi_2 & \dots & 1+\sin\varphi_n \\ \sin\varphi_1+\sin^2\varphi_1 & \sin\varphi_2+\sin^2\varphi_2 & \dots & \sin\varphi_n+\sin^2\varphi_n \\ \dots & \dots & \dots & \dots \\ \sin^{n-2}\varphi_1+\sin^{n-1}\varphi_1 & \sin^{n-2}\varphi_2+\sin^{n-1}\varphi_2 & \dots & \sin^{n-2}\varphi_n+\sin^{n-1}\varphi_n \end{vmatrix}.$$

267. $$\begin{vmatrix} 1 & 1 & \dots & 1 \\ \varphi_1(x_1) & \varphi_1(x_2) & \dots & \varphi_1(x_n) \\ \varphi_2(x_1) & \varphi_2(x_2) & \dots & \varphi_2(x_n) \\ \dots & \dots & \dots & \dots \\ \varphi_{n-1}(x_1) & \varphi_{n-1}(x_2) & \dots & \varphi_{n-1}(x_n) \end{vmatrix}$$

where $\varphi_k(x)=x^k+a_{1k}x^{k-1}+\dots+a_{kk}$.

268. $$\begin{vmatrix} 1 & 1 & \dots & 1 \\ F_1(\cos\varphi_1) & F_1(\cos\varphi_2) & \dots & F_1(\cos\varphi_n) \\ F_2(\cos\varphi_1) & F_2(\cos\varphi_2) & \dots & F_2(\cos\varphi_n) \\ \dots & \dots & \dots & \dots \\ F_{n-1}(\cos\varphi_1) & F_{n-1}(\cos\varphi_2) & \dots & F_{n-1}(\cos\varphi_n) \end{vmatrix}$$

where $F_k(x) = a_{0k}x^k + a_{1k}x^{k-1} + \dots + a_{kk}$.

***269.**
$$\begin{vmatrix} 1 & 1 & \dots & 1 \\ \binom{x_1}{1} & \binom{x_2}{1} & \dots & \binom{x_n}{1} \\ \binom{x_1}{2} & \binom{x_2}{2} & \dots & \binom{x_n}{2} \\ \dots & \dots & \dots & \dots \\ \binom{x_1}{n-1} & \binom{x_2}{n-1} & \dots & \binom{x_n}{n-1} \end{vmatrix}$$

where $\binom{x}{k} = \frac{x(x-1)\dots(x-k+1)}{1\cdot 2\dots k}$.

***270.** Prove that the value of the determinant

$$\begin{vmatrix} 1 & a_1 & a_1^2 & \dots & a_1^{n-1} \\ 1 & a_2 & a_2^2 & \dots & a_2^{n-1} \\ \dots & \dots & \dots & \dots & \dots \\ 1 & a_n & a_n^2 & \dots & a_n^{n-1} \end{vmatrix}$$

is divisible by $1^{n-1}2^{n-2}\dots(n-1)$ for integral $a_1, a_2, \dots, a_n$.

Compute the determinants:

***271.**
$$\begin{vmatrix} 1 & 2 & 3 & \dots & n \\ 1 & 2^3 & 3^3 & \dots & n^3 \\ \dots & \dots & \dots & \dots & \dots \\ 1 & 2^{2n-1} & 3^{2n-1} & \dots & n^{2n-1} \end{vmatrix}.$$

***272.**
$$\begin{vmatrix} \frac{x_1}{x_1-1} & \frac{x_2}{x_2-1} & \dots & \frac{x_n}{x_n-1} \\ x_1 & x_2 & \dots & x_n \\ x_1^2 & x_2^2 & \dots & x_n^2 \\ \dots & \dots & \dots & \dots \\ x_1^{n-1} & x_2^{n-1} & \dots & x_n^{n-1} \end{vmatrix}.$$

***273.**
$$\begin{vmatrix} a_1^n & a_1^{n-1}b_1 & a_1^{n-2}b_1^2 & \dots & a_1b_1^{n-1} & b_1^n \\ a_2^n & a_2^{n-1}b_2 & a_2^{n-2}b_2^2 & \dots & a_2b_2^{n-1} & b_2^n \\ \dots & \dots & \dots & \dots & \dots & \dots \\ a_{n+1}^n & a_{n+1}^{n-1}b_{n+1} & a_{n+1}^{n-2}b_{n+1}^2 & \dots & a_{n+1}b_{n+1}^{n-1} & b_{n+1}^n \end{vmatrix}.$$

274.
$$\begin{vmatrix} \sin^{n-1}\alpha_1 & \sin^{n-2}\alpha_1\cos\alpha_1 & \dots & \sin\alpha_1\cos^{n-2}\alpha_1 & \cos^{n-1}\alpha_1 \\ \text{in}^{n-1}\alpha_2 & \sin^{n-2}\alpha_2\cos\alpha_2 & \dots & \sin\alpha_2\cos^{n-2}\alpha_2 & \cos^{n-1}\alpha_2 \\ \dots & \dots & \dots & \dots & \dots \\ \sin^{n-1}\alpha_n & \sin^{n-2}\alpha_n\cos\alpha_n & \dots & \sin\alpha_n\cos^{n-2}\alpha_n & \cos^{n-1}\alpha_n \end{vmatrix}.$$

***275.**

$$\begin{vmatrix} a_1^{2n}+1 & a_1^{2n-1}+a_1 & a_1^{2n-2}+a_1^2 & \dots & a_1^{n+1}+a_1^{n-1} & a_1^n \\ a_2^{2n}+1 & a_2^{2n-1}+a_2 & a_2^{2n-2}+a_2^2 & \dots & a_2^{n+1}+a_2^{n-1} & a_2^n \\ \dots & \dots & \dots & \dots & \dots & \dots \\ a_{n+1}^{2n}+1 & a_{n+1}^{2n-1}+a_{n+1} & a_{n+1}^{2n-2}+a_{n+1}^2 & \dots & a_{n+1}^{n+1}+a_{n+1}^{n-1} & a_{n+1}^n \end{vmatrix}.$$

***276.** $$\begin{vmatrix} 1 & \cos\varphi_0 & \cos 2\varphi_0 & \dots & \cos(n-1)\varphi_0 \\ 1 & \cos\varphi_1 & \cos 2\varphi_1 & \dots & \cos(n-1)\varphi_1 \\ \dots & \dots & \dots & \dots & \dots \\ 1 & \cos\varphi_{n-1} & \cos 2\varphi_{n-1} & \dots & \cos(n-1)\varphi_{n-1} \end{vmatrix}.$$

***277.** $$\begin{vmatrix} \sin(n+1)\alpha_0 & \sin n\alpha_0 & \dots & \sin\alpha_0 \\ \sin(n+1)\alpha_1 & \sin n\alpha_1 & \dots & \sin\alpha_1 \\ \dots & \dots & \dots & \dots \\ \sin(n+1)\alpha_n & \sin n\alpha_n & \dots & \sin\alpha_n \end{vmatrix}.$$

***278.** $$\begin{vmatrix} 1 & 1 & \dots & 1 \\ x_1(x_1-1) & x_2(x_2-1) & \dots & x_n(x_n-1) \\ x_1^2(x_1-1) & x_2^2(x_2-1) & \dots & x_n^2(x_n-1) \\ \dots & \dots & \dots & \dots \\ x_1^{n-1}(x_1-1) & x_2^{n-1}(x_2-1) & \dots & x_n^{n-1}(x_n-1) \end{vmatrix}.$$

***279.** $$\begin{vmatrix} 1 & 1 & \dots & 1 \\ x_1^2 & x_2^2 & \dots & x_n^2 \\ x_1^3 & x_2^3 & \dots & x_n^3 \\ \dots & \dots & \dots & \dots \\ x_1^n & x_2^n & \dots & x_n^n \end{vmatrix}.$$

***280.** $$\begin{vmatrix} 1 & 1 & \dots & 1 \\ x_1 & x_2 & \dots & x_n \\ x_1^2 & x_2^2 & \dots & x_n^2 \\ \dots & \dots & \dots & \dots \\ x_1^{n-2} & x_2^{n-2} & \dots & x_n^{n-2} \\ x_1^n & x_2^n & \dots & x_n^n \end{vmatrix}.$$

281.

$$\begin{vmatrix} 1 & 1 & \dots & 1 \\ x_1 & x_2 & \dots & x_n \\ x_1^2 & x_2^2 & \dots & x_n^2 \\ \dots & \dots & \dots & \dots \\ x_1^{s-1} & x_2^{s-1} & \dots & x_n^{s-1} \\ x_1^{s+1} & x_2^{s+1} & \dots & x_n^{s+1} \\ \dots & \dots & \dots & \dots \\ x_1^n & x_2^n & \dots & x_n^n \end{vmatrix}.$$

***282.**

$$\begin{vmatrix} 1+x_1 & 1+x_1^2 & \dots & 1+x_1^n \\ 1+x_2 & 1+x_2^2 & \dots & 1+x_2^n \\ \dots & \dots & \dots & \dots \\ 1+x_n & 1+x_n^2 & \dots & 1+x_n^n \end{vmatrix}.$$

283.

$$\begin{vmatrix} 1 & x & x^2 & x^3 \\ x^3 & x^2 & x & 1 \\ 1 & 2x & 3x^2 & 4x^3 \\ 4x^3 & 3x^2 & 2x & 1 \end{vmatrix}.$$

284.

$$\begin{vmatrix} 1 & x & x^2 & x^3 & x^4 \\ 1 & 2x & 3x^2 & 4x^3 & 5x^4 \\ 1 & 4x & 9x^2 & 16x^3 & 25x^4 \\ 1 & y & y^2 & y^3 & y^4 \\ 1 & 2y & 3y^2 & 4y^3 & 5y^4 \end{vmatrix}.$$

***285.**

$$\begin{vmatrix} 1 & x & x^2 & \dots & x^n \\ 1 & 2x & 3x^2 & \dots & (n+1)x^n \\ 1 & 2^2x & 3^2x^2 & \dots & (n+1)^2x^n \\ \dots & \dots & \dots & \dots & \dots \\ 1 & 2^{n-1}x & 3^{n-1}x^2 & \dots & (n+1)^{n-1}x^n \\ 1 & y & y^2 & \dots & y^n \end{vmatrix}.$$

***286.**

$$\begin{vmatrix} 1 & x & x^2 & \dots & x^{n-1} \\ 1 & 2x & 3x^2 & \dots & nx^{n-1} \\ 1 & 2^2x & 3^2x^2 & \dots & n^2x^{n-1} \\ \dots & \dots & \dots & \dots & \dots \\ 1 & 2^{k-1}x & 3^{k-1}x^2 & \dots & n^{k-1}x^{n-1} \\ 1 & y_1 & y_1^2 & \dots & y_1^{n-1} \\ 1 & y_2 & y_2^2 & \dots & y_2^{n-1} \\ \dots & \dots & \dots & \dots & \dots \\ 1 & y_{n-k} & y_{n-k}^2 & \dots & y_{n-k}^{n-1} \end{vmatrix}.$$

***287.**

$$\begin{vmatrix} 1 & x & x^2 & \dots & x^{n-1} \\ 0 & 1 & C_2^1x & \dots & C_{n-1}^1x^{n-2} \\ 0 & 0 & 1 & \dots & C_{n-1}^2x^{n-3} \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & C_{n-1}^{k-1}x^{n-k} \\ 1 & y & y^2 & \dots & y^{n-1} \\ 0 & 1 & C_2^1y & \dots & C_{n-1}^1y^{n-2} \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & C_{n-1}^{n-k-1}y^k \end{vmatrix}.$$

288.

(a) Write the expansion of a fourth-order determinant in terms of the minors of the first two rows.