1. Whoever supports Ickes will vote for Jones. Anderson will vote for no one but a friend of Harris. No friend of Kelly has Jones for a friend. Therefore, if Harris is a friend of Kelly, Anderson will not support Ickes. (*Sxy-x* supports *y*, *Vxy-x* votes for *y*, *Fxy-x* is a friend of *y*, *a*-Anderson, *i*-Ickes, *j*-Jones, *h*-Harris, *k*-Kelly.)
2. Whoever belongs to the Country Club is wealthier than any member of the Elks Lodge. Not all who belong to the Country Club are wealthier than all who do not belong. Therefore not everyone belongs either to the Country Club or the Elks Lodge. (*Cx-x* belongs to the Country Club, *Ex-x* belongs to the Elks Lodge, *Px-x* is a person, *Wxy-x* is wealthier than *y*.)
3. All circles are figures. Therefore all who draw circles draw figures. (*Cx-x* is a circle, *Fx-x* is a figure, *Dxy-x* draws *y*.)
4. There is a professor who is liked by every student who likes any professor at all. Every student likes some professor or other. Therefore there is a professor who is liked by all students. (*Px-x* is a professor, *Sx-x* is a student, *Lxy-x* likes *y*.)
5. Only a fool would lie about one of Bill's fraternity brothers to him. A classmate of Bill's lied about Al to him. Therefore if none of Bill's classmates are fools, then Al is not a fraternity brother of Bill. (*Fx-x* is a fool, *Lxyz-x* lies about *y* to *z*, *Cxy-x* is a classmate of *y*, *Bxy-x* is a fraternity brother of *y*, *a*-Al, *b*-Bill.)
6. It is a crime to sell an unregistered gun to anyone. All the weapons that Red owns were purchased by him from either Lefty or Moe. So if one of Red's weapons is an unregistered gun, then if Red never bought anything from Moe, Lefty is a criminal. (*Rx-x* is registered, *Gx-x* is a gun, *Cx-x* is a criminal, *Wx-x* is a weapon, *Oxy-x* owns *y*, *Sxyz-x* sells *y* to *z*, *r*-Red, *l*-Lefty, *m*-Moe.)
7. No one respects a person who does not respect himself. No one will hire a person he does not respect. Therefore a person who respects no one will never be hired by anybody. (*Px-x* is a person, *Rxy-x* respects *y*, *Hxy-x* hires *y*.)
8. Everything on my desk is a masterpiece. Anyone who writes a masterpiece is a genius. Someone very obscure wrote some of the novels on my desk. Therefore some very obscure person is a genius. (*Dx-x* is on my desk, *Mx-x* is a masterpiece, *Px-x* is a person, *Gx-x* is a genius, *Ox-x* is very obscure, *Nx-x* is a novel, *Wxy-x* wrote *y*.)
9. Any book which is approved by all critics is read by every literary person. Anyone who reads anything will talk about it. A critic will approve any book written by any person who flatters him. Therefore if someone flatters every critic then any book he writes will be talked about by all literary persons. (*Bx-x* is a book, *Cx-x* is a critic, *Lx-x* is literary, *Px-x* is a person, *Axy-x* approves *y*, *Rxy-x* reads *y*, *Txy-x* talks about *y*, *Fxy-x* flatters *y*, *Wxy-x* writes *y*.)

10. A work of art which tells a story can be understood by everyone. Some religious works of art have been created by great artists. Every religious work of art tells an inspirational story. Therefore if some people admire only what they cannot understand, then some artists' creations will not be admired by everyone. (*Ax-x* is an artist, *Gx-x* is great, *Px-x* is a person, *Sx-x* is a story, *Ix-x* is inspirational, *Rx-x* is religious, *Wx-x* is a work of art, *Cxy-x* creates *y*, *Axy-x* admires *y*, *Txy-x* tells *y*, *Uxy-x* can understand *y*.)

III. SOME PROPERTIES OF RELATIONS

There are a number of interesting properties that relations themselves may possess. We shall consider only a few of the more familiar ones, and our discussion will be confined to properties of *dyadic* relations.

Dyadic relations may be characterized as *symmetrical*, *asymmetrical*, or *non-symmetrical*. Various symmetrical relations are designated by the phrases: 'is next to', 'is married to', and 'has the same weight as'. A *symmetrical* relation is one such that if one individual has that relation to a second individual, then the second individual must have that relation to the first. A propositional function '*Rxy*' designates a symmetrical relation if and only if

$$(x)(y)(Rxy \supset Ryx).$$

On the other hand, an *asymmetrical* relation is one such that if one individual has that relation to a second individual, then the second individual *cannot* have that relation to the first. Various asymmetrical relations are designated by the phrases: 'is north of', 'is parent of', and 'weighs more than'. A propositional function '*Rxy*' designates an asymmetrical relation if and only if

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$$(x)(y)(Rxy \supset Ryx).$$

On the other hand, an *asymmetrical* relation is one such that if one individual has that relation to a second individual, then the second individual *cannot* have that relation to the first. Various asymmetrical relations are designated by the phrases: 'is north of', 'is parent of', and 'weighs more than'. A propositional function '*Rxy*' designates an asymmetrical relation if and only if

$$(x)(y)(Rxy \supset \sim Ryx).$$

Not all relations are either symmetrical or asymmetrical, however. If one individual loves a second, or is a brother of a second, or weighs no more than a second, it does not follow that the second loves the first, or is a brother to the first (possibly being a sister instead), or weighs no more than the first. Nor does it follow that the second does *not* love the first, or is *not* a brother to him, or *does* weigh more than the first. Such relations as these are *non-symmetrical*, and are defined as those which are neither symmetrical nor asymmetrical.

Dyadic relations may also be characterized as *transitive*, *intransitive*, or *non-transitive*. Various transitive relations are designated by the phrases: 'is north of', 'is an ancestor of', and 'weighs the same as'. A *transitive* relation is one such that if one individual has it to a second, and the second to a third, then the first must have it to the third. A propositional function 'Rxy' designates a transitive relation if and only if

$$(x)(y)(z)[(Rxy \cdot Ryz) \supset Rxz].$$

An *intransitive* relation, on the other hand, is one such that if one individual has it to a second, and the second to a third, then the first *cannot* have it to the third. Some intransitive relations are designated by the phrases: 'is mother of', 'is father of', and 'weighs exactly twice as much as'. A propositional function 'Rxy' designates an intransitive relation if and only if

$$(x)(y)(z)[(Rxy \cdot Ryz) \supset \sim Rxz].$$

Not all relations are either transitive or intransitive. We define a *non-transitive* relation as one which is neither transitive nor intransitive; examples of non-transitive relations are designated by: 'loves', 'is discriminably different from', and 'has a different weight than'.

Finally, relations may be *reflexive*, *irreflexive*, or *non-reflexive*. Various definitions of these properties have been proposed by different authors, and there seems to be no standard terminology established. It is convenient to distinguish between reflexivity and total reflexivity. A relation is *totally reflexive* if every individual has that relation to itself. For example, the phrase 'is identical with' designates the totally reflexive relation of identity. A propositional function 'Rxy' designates a totally reflexive relation if and only if

$$(x)Rxx.$$

On the other hand, a relation is said to be *reflexive* if any individuals which stand in that relation to each other also have that relation to themselves. Obvious examples of reflexive relations are designated by the phrases: 'has the same color hair as', 'is the same age as', and 'is a contemporary of'. A propositional function 'Rxy' designates a reflexive relation if and only if

$$(x)(y)[(Rxy \supset (Rxx \cdot Ryy)].$$

It is obvious that all totally reflexive relations are reflexive.

An irreflexive relation is one which no individual has to itself. A propositional function 'Rxy' designates an irreflexive relation if and only if

$$(x) \sim Rxx.$$

Examples of irreflexive relations are common indeed; the phrases: 'is north of', 'is married to', and 'is parent of' all designate irreflexive relations. Relations which are neither reflexive nor irreflexive are said to be *non-reflexive*. The phrases: 'loves', 'hates', and 'criticizes' designate non-reflexive relations.

Relations may have various combinations of the properties described. The relation of *weighing more than* is asymmetrical, transitive, and irreflexive, while the relation of *having the same weight as* is symmetrical, transitive, and reflexive. However, some properties entail the presence of others. For example, all asymmetrical relations must be irreflexive, as can easily be demonstrated. Let 'Rxy' designate any asymmetrical relation; then by definition:

1. $(x)(y)(Rxy \supset \sim Ryx)$.

From this premiss we can deduce that R is irreflexive, that is, that $(x)\sim Rxx$:

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It is obvious that all totally reflexive relations are reflexive.

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1. $(x)(y)(Rxy \supset \sim Ryx)$.

From this premiss we can deduce that R is irreflexive, that is, that $(x)\sim Rxx$:

2. $(y)(Rxy \supset \sim Ryx)$	1, **UI**
3. $Rxx \supset \sim Rxx$	2, **UI**
4. $\sim Rxx \vee \sim Rxx$	3, Impl.
5. $\sim Rxx$	4, Taut.
6. $(x) \sim Rxx$	5, **UG**

Other logical connections among these properties of relations are easily stated and proved, but our interest lies in another direction.

The relevance of these properties to relational arguments is easily seen. An argument to which one of them is relevant might be stated thus:

> Tom has the same weight as Dick.
> Dick has the same weight as Harry.
> The relation of *having the same weight as* is transitive.
> Therefore Tom has the same weight as Harry.

When it is translated into our symbolism as

$$Wtd$$
$$Wdh$$
$$(x)(y)(z)[(Wxy \cdot Wyz) \supset Wxz]$$
$$\therefore Wth$$

the method of its validation is immediately obvious. We said that the argument 'might' be stated in the way indicated. But such a statement of the argument would be the rare exception rather than the rule. The ordinary way of propounding such an argument would be to state only the first two premisses and the conclusion, on the grounds that *everyone knows* that *having the same weight as* is a transitive relation. Relational arguments are often used, and many of them depend essentially on the transitivity, or symmetry, or one of the other properties of the relations involved. But *that* the relation in question *has* the relevant property is seldom—if ever—stated explicitly as a premiss. The reason is easy to see. In most discussions a large body of propositions can be presumed to be common knowledge. The majority of speakers and writers save themselves trouble

by not repeating well-known and perhaps trivially true propositions which their hearers or readers can perfectly well be expected to supply for themselves. An argument which is incompletely expressed, part of it being 'understood', is an *enthymeme.*

Because it is incomplete, an enthymeme must have its suppressed premiss or premisses taken into account when the problem arises of testing its validity. Where a necessary premiss is missing, the inference is technically invalid. But where the unexpressed premiss is easily supplied and obviously true, in all fairness it ought to be included as part of the argument in any evaluation of it. In such a case one assumes that the maker of the argument did have more 'in mind' than he stated explicitly. In most cases there is no difficulty in supplying the tacit premiss that the speaker intended but did not express. Thus the first specimen argument stated at the beginning of this chapter:

> Al is older than Bill.
> Bill is older than Charlie.
> Therefore Al is older than Charlie.

ought to be counted as valid, since it becomes so when the trivially true proposition that *being older than* is a transitive relation, is added as an auxiliary premiss. When the indicated missing premiss is supplied, a formal proof of the argument's validity is very easily set down.

Of course premisses other than relational ones are often left unexpressed. For example, in the argument

> Any horse can outrun any dog. Some greyhounds can outrun any rabbit. Therefore any horse can outrun any rabbit.

not only is the needed premiss about the transitivity of *being able to outrun* left unexpressed, but also the non-relational premiss that all greyhounds are dogs. When these are added—and they are certainly not debatable issues—the validity of the argument can be demonstrated as follows:

2. $(y)(Rxy \supset \sim Ryx)$	1, **UI**
3. $Rxx \supset \sim Rxx$	2, **UI**
4. $\sim Rxx \vee \sim Rxx$	3, Impl.
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1.	$(x)[Hx \supset (y)(Dy \supset Oxy)]$	premisses/ ∴ $(x)[Hx \supset (z)(Rz \supset Oxz)]$
2.	$(\exists y)[Gy \cdot (z)(Rz \supset Oyz)]$	
3.	$(x)(y)(z)[(Oxy \cdot Oyz) \supset Oxz]$	additional premisses
4.	$(y)(Gy \supset Dy)$	
→5.	Hx	
6.	$Hx \supset (y)(Dy \supset Oxy)$	1, **UI**
7.	$(y)(Dy \supset Oxy)$	6, 5, M.P.
8.	$Gy \cdot (z)(Rz \supset Oyz)$	2, **EI**
9.	Gy	8, Simp.
10.	$Gy \supset Dy$	4, **UI**
11.	Dy	10, 9, M.P.
12.	$Dy \supset Oxy$	7, **UI**
13.	Oxy	12, 11, M.P.
14.	$(z)(Rz \supset Oyz)$	8, Simp.
→15.	Rz	
16.	$Rz \supset Oyz$	14, **UI**
17.	Oyz	16, 15, M.P.
18.	$Oxy \cdot Oyz$	13, 17, Conj.
19.	$(y)(z)[(Oxy \cdot Oyz) \supset Oxz]$	3, **UI**
20.	$(z)[(Oxy \cdot Oyz) \supset Oxz]$	19, **UI**
21.	$(Oxy \cdot Oyz) \supset Oxz$	20, **UI**
22.	Oxz	21, 18, M.P.
23.	$Rz \supset Oxz$	15–22, C.P.
24.	$(z)(Rz \supset Oxz)$	23, **UG**
25.	$Hx \supset (z)(Rz \supset Oxz)$	5–24, C.P.
26.	$(x)[Hx \supset (z)(Rz \supset Oxz)]$	25, **UG**

Missing premisses are not always so easily noticed and supplied as in the present example. When it is not so obvious which necessary premisses are missing from an enthymematically expressed argument, then in beginning a proof of its validity it is a good policy to leave a little space just below the given premisses, in which additional premisses can be written when need arises for their use. The only point to be stressed is that no statement which is as doubtful or debatable as the argument's own conclusion is to be admitted as a supplementary premiss, for in a valid argument which is enthymematically stated only the sheerest platitudes should be left unexpressed for the hearer or reader to fill in for himself.

EXERCISES

Prove the validity of the following enthymemes—adding only obviously true premises where necessary:

1. A Cadillac is more expensive than any low-priced car. Therefore no Cadillac is a low-priced car. (*Cx*-*x* is a Cadillac, *Lx*-*x* is a low-priced car, *Mxy*-*x* is more expensive than *y*.)
2. Alice is Betty's mother. Betty is Charlene's mother. Therefore if Charlene loves only her mother then she does not love Alice. (*a*-Alice, *b*-Betty, *c*-Charlene, *Mxy*-*x* is mother of *y*, *Lxy*-*x* loves *y*.)
3. Any man on the first team can outrun every man on the second team. Therefore no man on the second team can outrun any man on the first team. (*Fx*-*x* is a man on the first team, *Sx*-*x* is a man on the second team, *Oxy*-*x* can outrun *y*.)
4. Every boy at the party danced with every girl who was there. Therefore every girl at the party danced with every boy who was there. (*Bx*-*x* is a boy, *Gx*-*x* is a girl, *Px*-*x* was at the party, *Dxy*-*x* danced with *y*.)
5. Anyone is unfortunate who bears the same name as a person who commits a crime. Therefore anyone who commits a burglary is unfortunate. (*Px*-*x* is a person, *Ux*-*x* is unfortunate, *Cx*-*x* is a crime, *Bx*-*x* is a burglary, *Cxy*-*x* commits *y*, *Nxy*-*x* bears the same name as *y*.)
6. All the watches sold by Kubitz are made in Switzerland. Anything made in a foreign country has a tariff paid on it. Anything on which a tariff was paid costs its purchaser extra. Therefore it will cost anyone extra who buys a watch from Kubitz. (*Wx*-*x* is a watch, *Tx*-*x* has a tariff paid on it, *Fx*-*x* is a foreign country, *Cxy*-*x* costs *y* extra, *Mxy*-*x* is made in *y*, *Bxyz*-*x* buys *y* from *z*, *s*-Switzerland, *k*-Kubitz.)
7. Vacant lots provide no income to their owners. Anyone who owns real estate must pay taxes on it. Therefore anyone who owns a vacant lot must pay taxes on something which provides no income to him. (*Vx*-*x* is a vacant lot, *Rx*-*x* is real estate, *Ixy*-*x* provides income to *y*, *Txy*-*x* pays taxes on *y*, *Oxy*-*x* owns *y*.)
8. All admirals wear uniforms having gold buttons. Therefore some naval officers wear clothes which have metal buttons. (*Ax*-*x* is an admiral, *Ux*-*x* is a uniform, *Gx*-*x* is gold, *Bx*-*x* is a button, *Nx*-*x* is a naval officer, *Cx*-*x* is clothing, *Mx*-*x* is metal, *Wxy*-*x* wears *y*, *Hxy*-*x* has *y*.)

9. Whenever Charlie moved to Boston, it was after he had met Al. Whenever Charlie got married, it was before he ever saw Dave. Therefore if Charlie moved to Boston and subsequently got married, then he met Al before he ever saw Dave. (Tx-x is a time, Ax-Charlie met Al at (time) x, Bx-Charlie moved to Boston at (time) x, Mx-Charlie got married at (time) x, Dx-Charlie saw Dave at (time) x, Pxy-x precedes y.)
10. A fish that chases every shiner will be hooked by an angler who uses a shiner for bait. A greedy fish will chase every shiner. So if all anglers are sportsmen, then no pike which is not hooked by a sportsman who uses minnows for bait is greedy. (Fx-x is a fish, Sx-x is a shiner, Cxy-x chases y, Hxy-x hooks y, Ax-x is an angler, Bxy-x uses y for bait, Gx-x is greedy, Px-x is a pike, Rx-x is a sportsman, Mx-x is a minnow.)

IV. IDENTITY AND THE DEFINITE DESCRIPTION

The notion of *identity* is a familiar one. Perhaps the most natural occasion for its use is in the process of *identification*, as when in a police line-up a witness identifies a suspect, asserting that

> The man on the right *is* the man who snatched my purse.

Other identifications are common, as in a geography class when it is asserted that

> Mt. Everest *is* the tallest mountain in the world.

or when in a literature class it is asserted that

> Scott *is* the author of *Waverley*.

A relationship is asserted by each of the preceding propositions to hold between the individuals denoted by its two terms. The relation asserted to hold is that of *identity*. In each of the preceding at least one term was a *definite description*, which is a phrase of the form 'the so-and-so'. In identifications, however, both terms may be proper names. Just as the two propositions

> Brutus killed Caesar.

and

> Booth killed Lincoln.

assert the relation of *killing* to hold between the individuals denoted by the proper names appearing in them, so the propositions

> Lewis Carroll was Charles Lutwidge Dodgson.

and

> Mark Twain was Samuel Clemens.

assert the relation of *identity* to hold between the individuals denoted by the proper names appearing in them.

The usual notation for the relation of identity is the ordinary equals-sign '='. It is intuitively obvious that the relation of identity is transitive, symmetrical, and totally reflexive. In our symbolic notation we can write

$$(x)(y)(z)\{[(x = y)\cdot(y = z)] \supset (x = z)\}$$
$$(x)(y)[(x = y) \supset (y = x)]$$
$$(x)(x = x).$$

All of these are immediate consequences of the definition of identity contained in Leibniz's principle of the Identity of Indiscernibles:

> $x = y$ if and only if every property of x is a property of y, and conversely.

This principle permits us to infer, from the premisses $\nu = \mu$ and any proposition containing an occurrence of the symbol ν, as conclusion any proposition which results from replacing any occurrences of ν in the second premiss by the symbol μ. Any inference of this pattern is valid, and in a proof should have the letters 'Id.' written beside it. A specimen deduction or two will make this clear. The argument

> O. Henry was William Sidney Porter.
> O. Henry was a writer.
> ———
> Therefore William Sidney Porter was a writer.