10.3. We start with the equation

$$\lambda_1 x_1^2 + \ldots + \lambda_r x_r^2 + \mu x_{r+1} = 0,$$

where all coefficients are nonzero.

By multiplication by scalars and rearranging positive and negative coefficients, we can bring it to the form

$$\frac{x_1^2}{p_1} + \ldots + \frac{x_k^2}{p_k} - \frac{x_{k+1}^2}{p_{k+1}} - \ldots - \frac{x_r^2}{p_r} = 2x_{r+1},$$

where $p_i > 0$ $(1 \le i \le r)$, $1 \le r \le n-1$,

$k \geq \frac{r}{2}$ (if r is even), $k \geq \frac{r+1}{2}$ (if r is odd)

- If $r = n-1$, we call the surface <u>$(n-1)$-dimensional paraboloid</u>.
- If $r < n-1$, we call it <u>cylinder</u> (over the corresponding $(r-1)$-dimensional paraboloid).

(optional)

- If $k = r = n-1$, we call it <u>$(n-1)$-dimensional elliptic paraboloid</u>.
- If $k < r = n-1$, we call it <u>$(n-1)$-dimensional hyperbolic paraboloid</u>.

11. $n=2$.

Nondegenerate ~~two-dimensional~~ one-dimensional quadrics (conics) are:

- ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$
- hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$
- parabola $x^2=2py$

$n=3$

Nondegenerate two-dimensional quadrics are

- ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$
- hyperboloids:

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1 \quad \text{(of one sheet)}$$

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}-\frac{z^2}{c^2}=1 \quad \text{(of two sheets)}$$

- paraboloids:

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=2z \quad \text{(elliptic)}$$

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=2z \quad \text{(hyperbolic)}$$