אנליזה 1 למורים- פתרון תרגיל 1-חזקות ולוגריתמים

פתרון שאלה 1

מצאו את x במשוואות הבאות:

$$3^{2x} - 4 \cdot 3^x + 3 = 0$$
$$t = 3^x$$
$$t^2 - 4t + 3 = 0$$
$$(t-3)(t-1) = 0 \qquad 1.$$
$$t = 3 : 3^x = 3 \Rightarrow x = 1$$
$$t = 1 : 3^x = 1 \Rightarrow x = 0$$

$$5 \cdot 3^x - 3^{x+1} = 162$$
$$5 \cdot 3^x - 3^x \cdot 3 = 162$$
$$2 \cdot 3^x = 162$$
$$3^x = 81 \qquad 2.$$
$$\log_3 81 = x$$
$$x = 4$$

$$\frac{20}{9^x + 1} = 3 - \frac{8}{9^x - 1}$$
$$t = 9^x$$
$$\frac{20}{t+1} = 3 - \frac{8}{t-1}$$
$$20(t-1) = 3(t-1)(t+1) - 8(t+1)$$
$$20t - 20 = 3(t^2 - 1) - 8t - 8$$
$$0 = 3t^2 - 28t + 9t \qquad 3.$$
$$t_{1,2} = \frac{28 \pm 26}{6}$$
$$t_1 = \frac{28 + 26}{6} = 9 \Rightarrow 9^x = 9 \Rightarrow x = 1$$
$$t_2 = \frac{28 - 26}{6} = \frac{1}{3} \Rightarrow 9^x = \frac{1}{3} \Rightarrow \log_9 1/3 = x \Rightarrow x = -0.5$$

פתרון שאלה 2

מצאו את x במשוואות הבאות:

$$\log_2(5^x + 3) = 7$$
$$2^7 = 5^x + 3$$
$$128 = 5^x + 3$$
$$125 = 5^x \qquad \text{1.}$$
$$\log_5 125 = x$$
$$x = 3$$

$$(\log_2 x)^2 + 2\log_2 x - 15 = 0$$
$$t = \log_2 x$$
$$t^2 + 2t - 15 = 0$$
$$(t-3)(t+5) = 0 \qquad \text{2.}$$
$$t = 3 : 3 = \log_2 x \Rightarrow 2^3 = x \Rightarrow x = 8$$
$$t = -5 : -5 = \log_2 x \Rightarrow 2^{-5} = x \Rightarrow x = \frac{1}{32}$$

$$\log_7 x - \frac{6}{\log_7 x} = 1$$
$$t = \log_7 x$$
$$t - \frac{6}{t} = 1$$
$$t^2 - 6 = t$$
$$t^2 - t - 6 = 0 \qquad \text{3.}$$
$$(t-3)(t+2) = 0$$
$$t = 3 : \log_7 x = 3 \Rightarrow 7^3 = x \Rightarrow x = 343$$
$$t = -2 : \log_7 x = -2 \Rightarrow 7^{-2} = x \Rightarrow x = \frac{1}{49}$$

$$(\log_2 x) \cdot \left(\log_2 \left(\frac{x}{8}\right)\right) = -2$$

$$(\log_2 x) \cdot (\log_2 x - \log_2 8) = -2$$

$$(\log_2 x) \cdot (\log_2 x - \log_2 8) = -2$$

$$t = \log_2 x$$

$$t(t-3) = -2$$

$$t^2 - 3t + 2 = 0 \qquad \qquad .4$$

$$(t-2)(t-1) = 0$$

$$t = 2 : 2 = \log_2 x \Rightarrow 2^2 = x \Rightarrow x = 4$$

$$t = 1 : 1 = \log_2 x \Rightarrow 2^1 = x \Rightarrow x = 2$$

$$\left(\log_3(3x)\right)^2 = \log_3(3x^2) + 1$$

$$\left(\log_3 3 + \log_3 x\right)^2 = \log_3 3 + \log_3 x^2 + 1$$

$$\left(1 + \log_3 x\right)^2 = 1 + 2\log_3 x + 1$$

$$t = \log_3 x$$

$$(1+t)^2 = 2 + 2t$$

$$t^2 + 2t + 1 = 2 + 2t \qquad \qquad .5$$

$$t^2 = 1$$

$$t = 1 : 1 = \log_3 x \Rightarrow x = 3$$

$$t = -1 : -1 = \log_3 x \Rightarrow x = \frac{1}{3}$$