תרגול 1

8.
$$\int \frac{e^{\tan(x)}\sin(x)}{\cos^3(x)}dx = \int \frac{e^{\tan(x)}\cdot\tan(x)}{\cos^2(x)}dx$$

$$\boxed{\begin{array}{l} t=\tan(x) \\ dt=\frac{dx}{\cos^2(x)} \end{array}}$$

$$= \int e^t\cdot t\,dt = e^t\cdot t - e^t + c =$$

$$= e^{\tan(x)}\tan(x) - e^{\tan(x)} + c$$

9.
$$\int e^{2x+e^x}dx = \int e^{2x}\cdot e^{e^x}dx = \int (e^x)^2\cdot e^{e^x}dx =$$

$$\boxed{\begin{array}{l} t=e^x \\ dt=e^x dx \end{array}}$$

$$= \int t^{*}\cdot e^t\,dt = e^t\cdot t - e^t + c =$$

$$= e^{e^x}\cdot e^x - e^{e^x} + c$$

פתרון לתרגילים מ-מבחנים:

1.
$$\int \frac{\sin(x)\cos(x)}{a^2\cos^2(x)+b^2\sin^2(x)}dx = \int \frac{\sin(x)\cos(x)dx}{a^2(1-\sin^2(x))+b^2\sin^2(x)} =$$

$$= \int \frac{\sin(x)\cos(x)dx}{a^2+(b^2-a^2)\sin^2(x)}dx = \int \frac{t\,dt}{a^2+(b^2-a^2)t^2} =$$

$$\boxed{\begin{array}{l} t=\sin(x) \\ dt=\cos(x)dx \end{array}}$$

$$= \frac{1}{2(b^2-a^2)}\int \frac{du}{u} = \frac{1}{2(b^2-a^2)}\ln|u| + c =$$

$$\boxed{\begin{array}{l} u=a^2+(b^2-a^2)t^2 \\ du=2(b^2-a^2)t\,dt \end{array}} \quad (b^2\neq a^2)$$

$$= \frac{1}{2(b^2-a^2)}\ln|a^2+(b^2-a^2)t^2| + c =$$

$$= \frac{1}{2(b^2-a^2)}\ln|a^2+(b^2-a^2)\sin^2(x)| + c$$

(*) אם $a^2=b^2$ אז נקבל:

$$\int \frac{\sin(x)\cos(x)}{a^2(\cos^2(x)+\sin^2(x))} = \frac{1}{2a^2}\int \sin(2x) = \frac{1}{2a^2}\left[-\frac{\cos(2x)}{2}\right] + c$$

2

$$\int \frac{3x\,dx}{\sqrt{9+x^2}} = \frac{3}{2}\int \frac{2x\,dx}{\sqrt{9+x^2}} = \frac{3}{2}\int \frac{dt}{\sqrt{t}} =$$

$$\boxed{\begin{aligned} t &= 9+x^2 \\ dt &= 2x\,dx \end{aligned}}$$

$$= \cancel{\frac{3}{2}}\, 3\sqrt{t} + C = 3\sqrt{9+x^2} + C$$

3

$$\int \frac{dx}{\sqrt{x}\,(1+\sqrt[3]{x})} = \int \frac{6t^{\cancel{5}\,2}\,dt}{\cancel{t^3}(1+t^2)} =$$

$$\boxed{\begin{aligned} t^6 &= x \\ \sqrt{x} &= t^3 \\ \sqrt[3]{x} &= t^2 \\ dx &= 6t^5\,dt \end{aligned}}$$

$$= \int \frac{6t^2}{1+t^2}\,dt = 6\int \frac{t^2+1-1}{1+t^2}\,dt =$$

$$= 6\left[\int dt - \int \frac{dt}{1+t^2}\right] =$$

$$= 6\left[t - \arctan(t)\right] + C =$$

$$= 6\left[\sqrt[6]{x} - \arctan(\sqrt[6]{x})\right] + C$$

4

$$\int \frac{\sin^3(x)}{\cos(x)}\,dx = \int \frac{(1-\cos^2(x))\cdot\sin(x)}{\cos(x)}\,dx =$$

$$\boxed{\begin{aligned} t &= \cos(x) \\ dt &= -\sin(x)\,dx \end{aligned}}$$

$$= -\int \frac{(1-t^2)\,dt}{t} =$$

$$= \int \frac{t^2-1}{t}\,dt = \int t - \int \frac{1}{t} =$$

$$= \frac{t^2}{2} - \ln|t| + C = \frac{\cos^2(x)}{2} - \ln|\cos(x)| + C$$

$$\int \frac{x\,dx}{\sqrt{x+1}} = \cancel{\int \frac{2t\,dt}{t}} \cancel{\int} \ldots$$ 5

$$\boxed{\begin{aligned} t^2 &= x+1 \\ x &= t^2-1 \end{aligned}}$$

$dx = 2t\,dt$

$$= \int \frac{(t^2-1)\cdot 2t\,dt}{\cancel{t}} = 2\int (t^2-1)\,dt =$$

$$= 2\left[\frac{t^3}{3} - t\right] + C = 2\left[\frac{(\sqrt{x+1})^3}{3} - \sqrt{x+1}\right] + C$$

6

$$\int \arctan\left(\frac{1}{x}\right) dx =$$

$\cancel{t = \frac{1}{x}}$

$\cancel{x = \frac{1}{t}}$

$\cancel{dx = -\frac{1}{t^2}\,dt}$

$$\boxed{\begin{aligned} f' &= 1 \qquad\qquad f = x \\ g &= \arctan\left(\tfrac{1}{x}\right) \qquad g' = \frac{1}{1+\left(\frac{1}{x^2}\right)}\cdot\left(-\frac{1}{x^2}\right) = \\ &\qquad = \frac{x^2}{x^2+1}\cdot\left(-\frac{1}{x^2}\right) = -\frac{1}{x^2+1} \end{aligned}}$$

$$= x\arctan\left(\frac{1}{x}\right) + \int \frac{x}{x^2+1}\,dx = x\arctan\left(\frac{1}{x}\right) + \frac{1}{2}\int \frac{dt}{t} =$$

$$\boxed{\begin{aligned} t &= x^2+1 \\ dt &= 2x\,dx \end{aligned}}$$

$$= x\arctan\left(\frac{1}{x}\right) + \frac{1}{2}\ln|t| + C =$$

$$= x\arctan\left(\frac{1}{x}\right) + \frac{1}{2}\ln(x^2+1) + C$$