1 Prove that an affine variety U is irreducible if and only if its projective closure $\overline{U}$ is irreducible.

2 Associate with any affine variety $U \subset \mathbb{A}^n_0$ its projective closure $\overline{U}$ in $\mathbb{P}^n$. Prove that this defines a one-to-one correspondence between the affine subvarieties of $\mathbb{A}^n_0$ and the projective subvarieties of $\mathbb{P}^n$ with no components contained in the hyperplane $S_0 = 0$.

3 Prove that the variety $X = \mathbb{A}^2 \setminus (0, 0)$ is not isomorphic to an affine variety. [Hint: Compute the ring $k[X]$ of regular functions on X, and use the fact that if Y is an affine variety, every proper ideal $\mathfrak{A} \subsetneqq k[Y]$ defines a nonempty set.]

4 Prove that any quasiprojective variety is open in its projective closure.

5 Prove that every rational map $\varphi: \mathbb{P}^1 \to \mathbb{P}^n$ is regular.

6 Prove that any regular map $\varphi: \mathbb{P}^1 \to \mathbb{A}^n$ maps $\mathbb{P}^1$ to a point.

7 Define a birational map f from an irreducible quadric hypersurface $X \subset \mathbb{P}^3$ to the plane $\mathbb{P}^2$ by analogy with the stereographic projection of Example 1.22. At which points is f not regular? At which points is f^{-1} not regular?

8 In Exercise 7, find the open subsets $U \subset X$ and $V \subset \mathbb{P}^2$ that are isomorphic.

9 Prove that the map $y_0 = x_1x_2$, $y_1 = x_0x_2$, $y_2 = x_0x_1$ defines a birational map of $\mathbb{P}^2$ to itself. At which points are f and f^{-1} not regular? What are the open sets mapped isomorphically by f? (Compare Section 3.5, Chapter 4.)

10 Prove that the Veronese image $v_m(\mathbb{P}^n) \subset \mathbb{P}^N$ is not contained in any linear subspace of $\mathbb{P}^N$.

11 Prove that the variety $\mathbb{P}^2 \setminus X$, where X is a plane conic, is affine. [Hint: Use the Veronese embedding.]