Exercise X

Problems:

1. Using induction, prove that any integer of the form $n^3+(n+1)^3+(n+2)^3$ is divided by 9.
2. Using induction, prove that $1+3+5+\cdots+(2n-1)=n^2$.
3. Using induction, prove that $1+4+7+\cdots+(3n-2)=2n(3n-1)$.
4. Using induction, prove that $\frac{1}{1\cdot 3}+\frac{1}{3\cdot 5}+\frac{1}{5\cdot 7}+\cdots+\frac{1}{(2n-1)\cdot(2n+1)}=\frac{1}{2n+1}$.
5. Using induction, prove that $1^3+2^3+3^3+\cdots+n^3=\left(\frac{n(n+1)}{2}\right)^2$.
6. Using induction, prove that $2n+1<2^n$ for $n\geqslant 3$.