

In[7]:= (\* Infi 4 - Problem set 1 - Solutions \*)

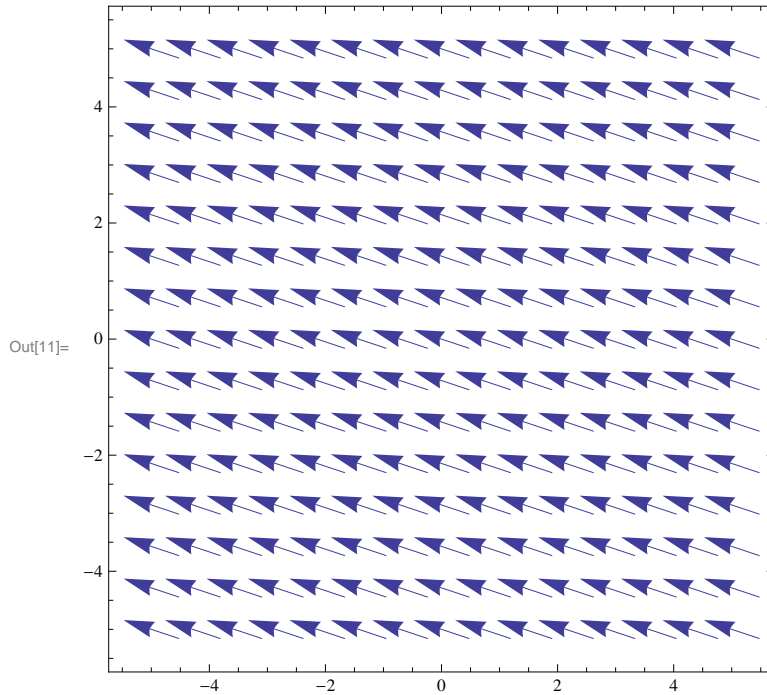
In[8]:= (\* question 1 \*)

In[16]:= (\* 1 \*)

In[9]:=  $\mathbf{F} = \{-3, 1\}$

Out[9]=  $\{-3, 1\}$

In[11]:= `VectorPlot[F, {x, -5, 5}, {y, -5, 5}]`

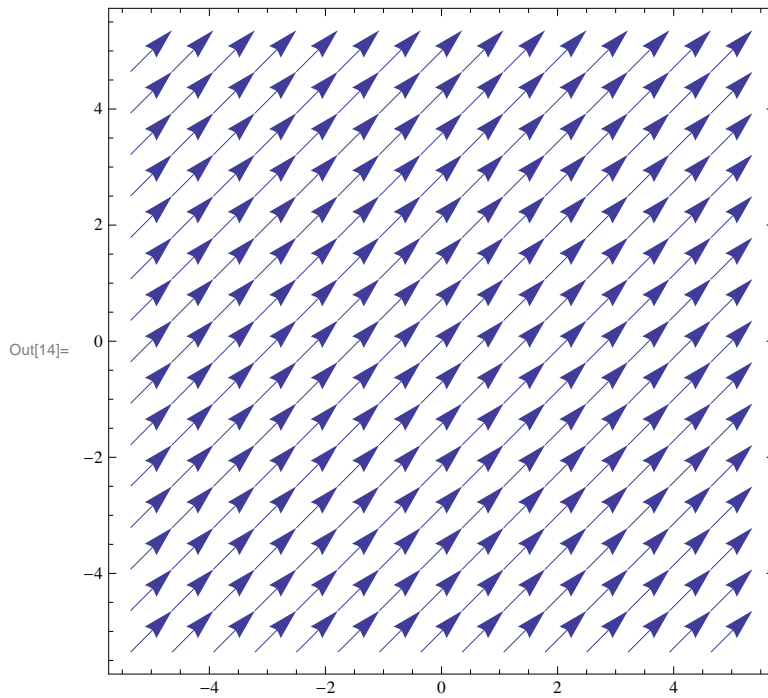


In[17]:= (\* 2 \*)

In[13]:=  $\mathbf{F} = \{1, 1\}$

Out[13]=  $\{1, 1\}$

```
In[14]:= VectorPlot[F, {x, -5, 5}, {y, -5, 5}]
```

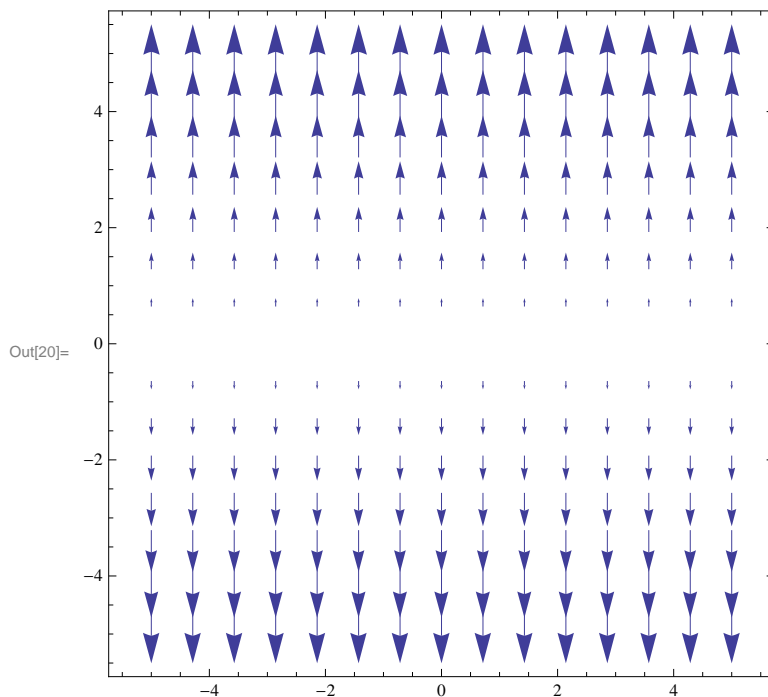


```
In[18]:= (* 3 *)
```

```
In[19]:= F = {0, y}
```

Out[19]= {0, y}

```
In[20]:= VectorPlot[F, {x, -5, 5}, {y, -5, 5}]
```

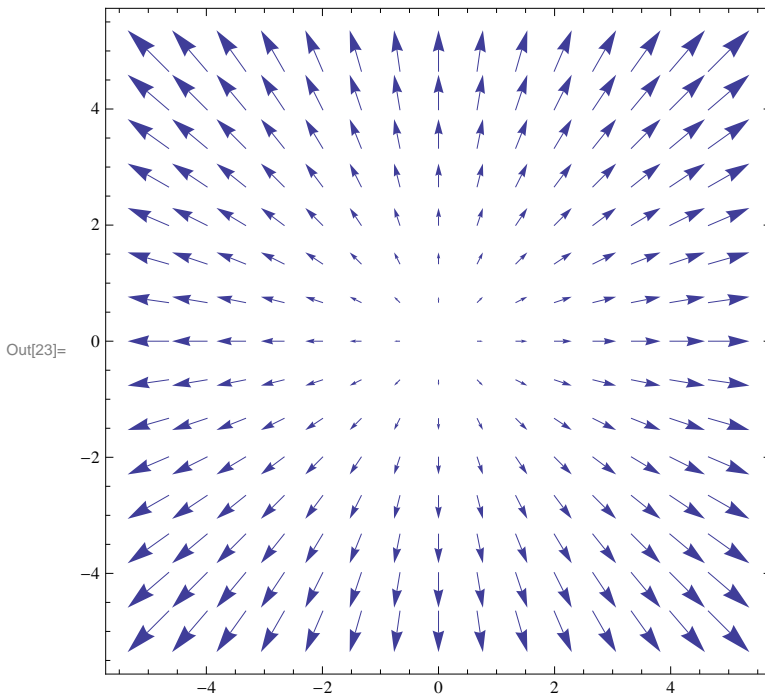


```
In[21]:= (* 4 *)
```

```
In[22]:= F = {x, y}
```

Out[22]= {x, y}

In[23]:= `VectorPlot[F, {x, -5, 5}, {y, -5, 5}]`

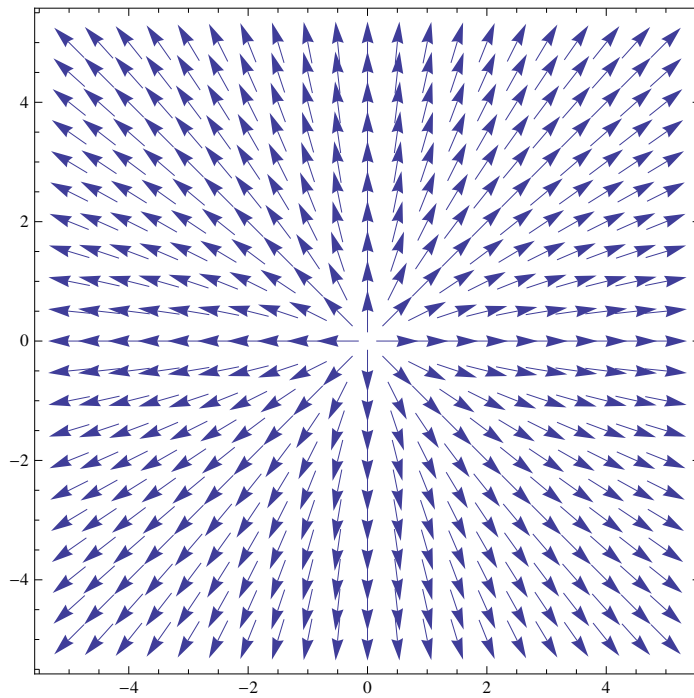


In[24]:= `(* 5 *)`

In[25]:= 
$$\mathbf{F} = \left\{ \frac{x}{\sqrt{x^2 + y^2}}, \frac{y}{\sqrt{x^2 + y^2}} \right\}$$

Out[25]= 
$$\left\{ \frac{x}{\sqrt{x^2 + y^2}}, \frac{y}{\sqrt{x^2 + y^2}} \right\}$$

In[29]:= `VectorPlot[F, {x, -5, 5}, {y, -5, 5}, VectorPoints -> {21, 21}]`

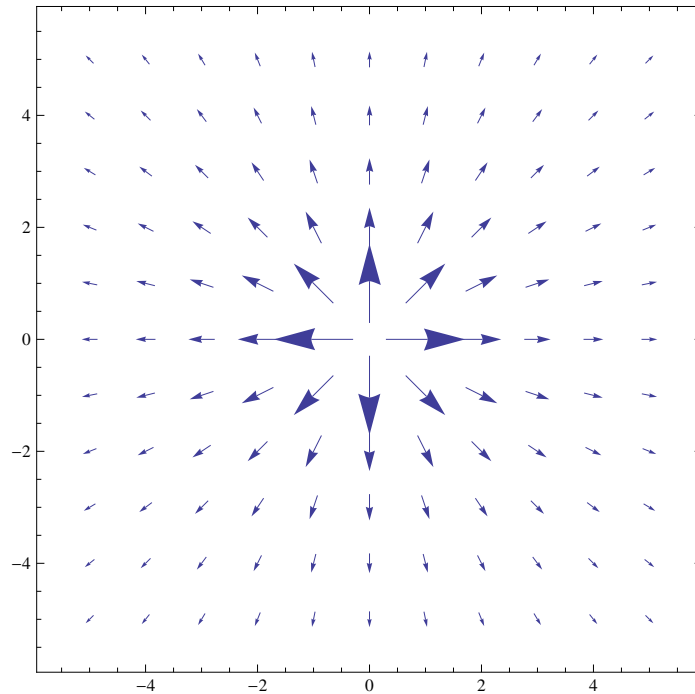


In[30]:= `(* 6 *)`

$$\text{In[31]:= } \mathbf{F} = \left\{ \frac{x}{x^2 + y^2}, \frac{y}{x^2 + y^2} \right\}$$

$$\text{Out[31]= } \left\{ \frac{x}{x^2 + y^2}, \frac{y}{x^2 + y^2} \right\}$$

**In[36]:= VectorPlot[F, {x, -5, 5}, {y, -5, 5}, VectorPoints -> {11, 11}]**

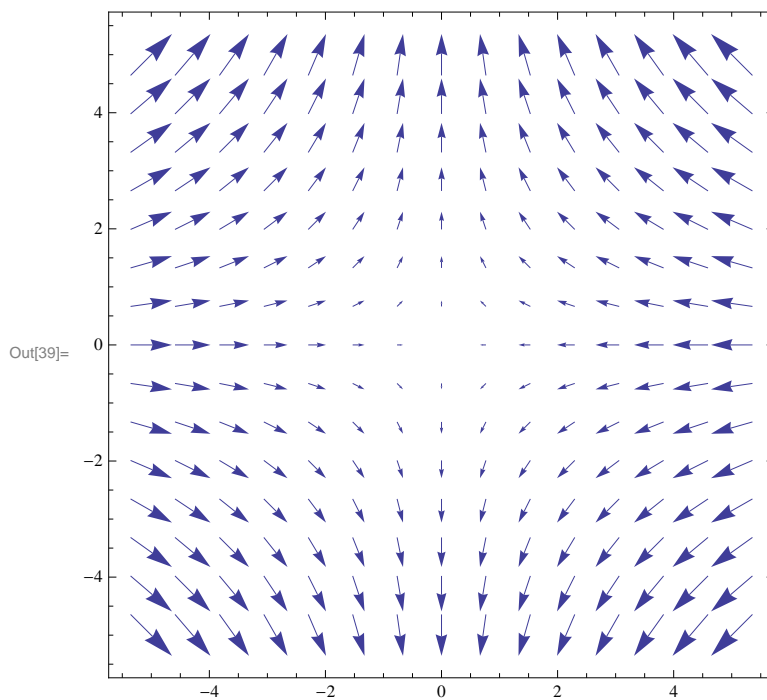


**In[37]:= (\* 7 \*)**

**In[38]:= F = {-x, y}**

**Out[38]= {-x, y}**

**In[39]:= VectorPlot[F, {x, -5, 5}, {y, -5, 5}]**

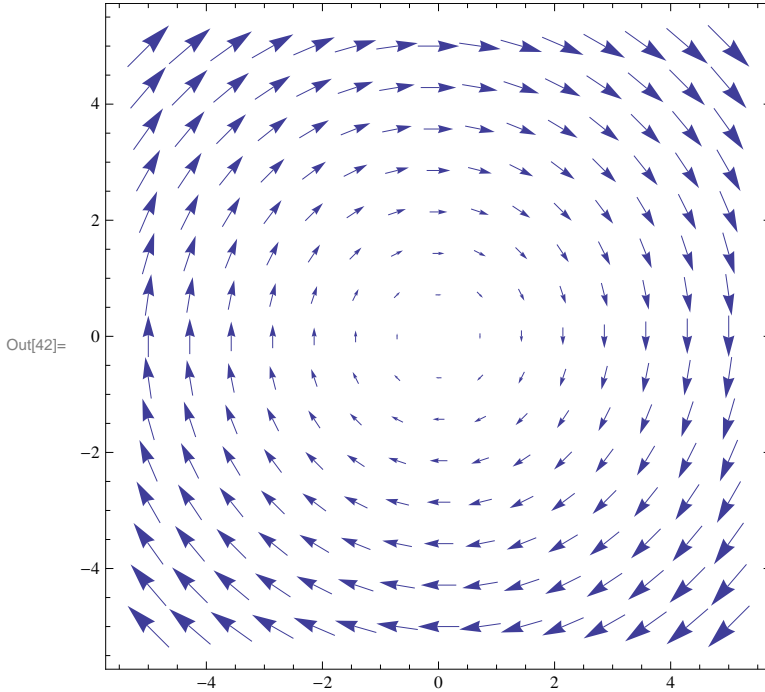


In[40]:= (\* 8 \*)

In[41]:=  $\mathbf{F} = \{2y, -2x\}$

Out[41]:=  $\{2y, -2x\}$

VectorPlot[F, {x, -5, 5}, {y, -5, 5}]

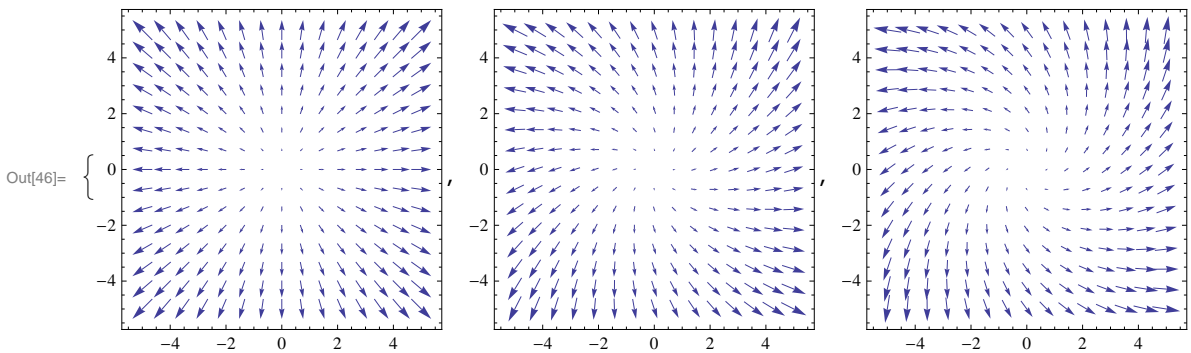


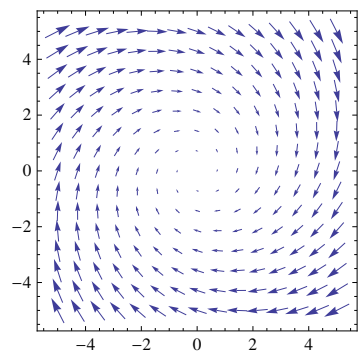
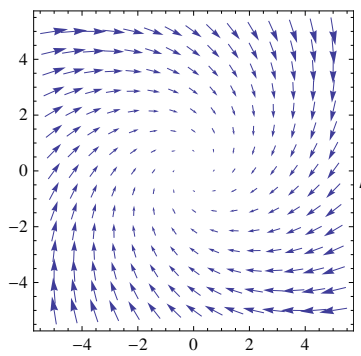
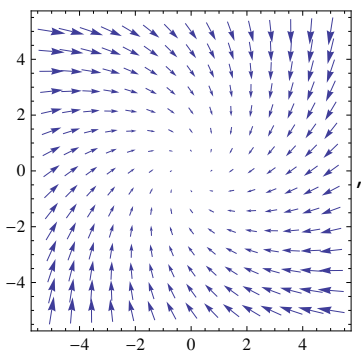
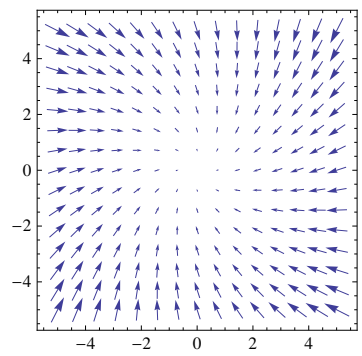
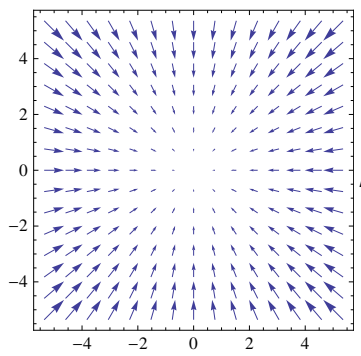
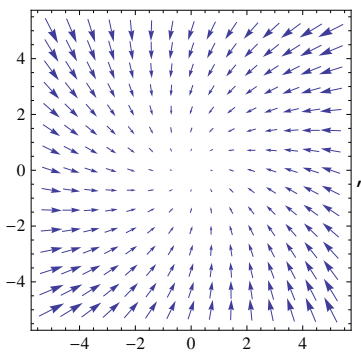
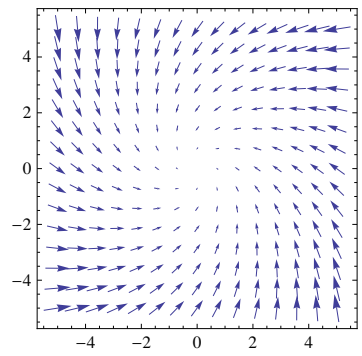
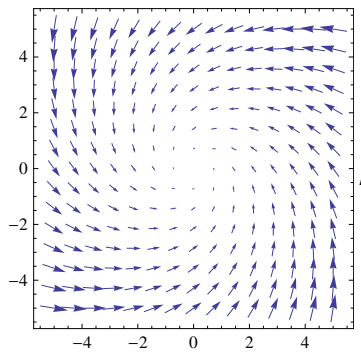
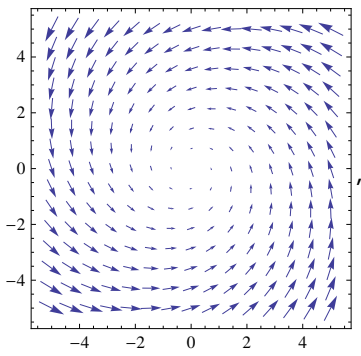
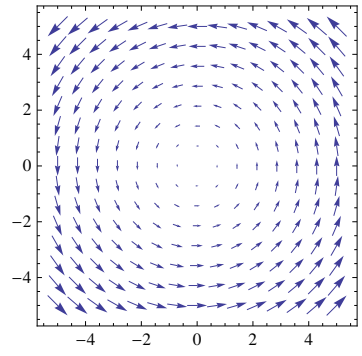
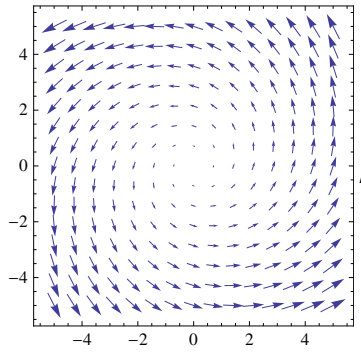
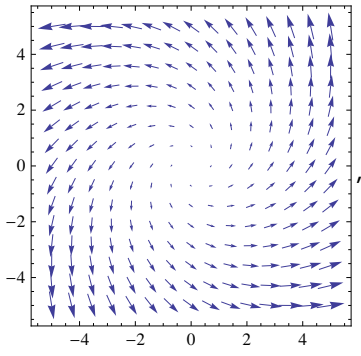
In[43]:= (\* 9 \*)

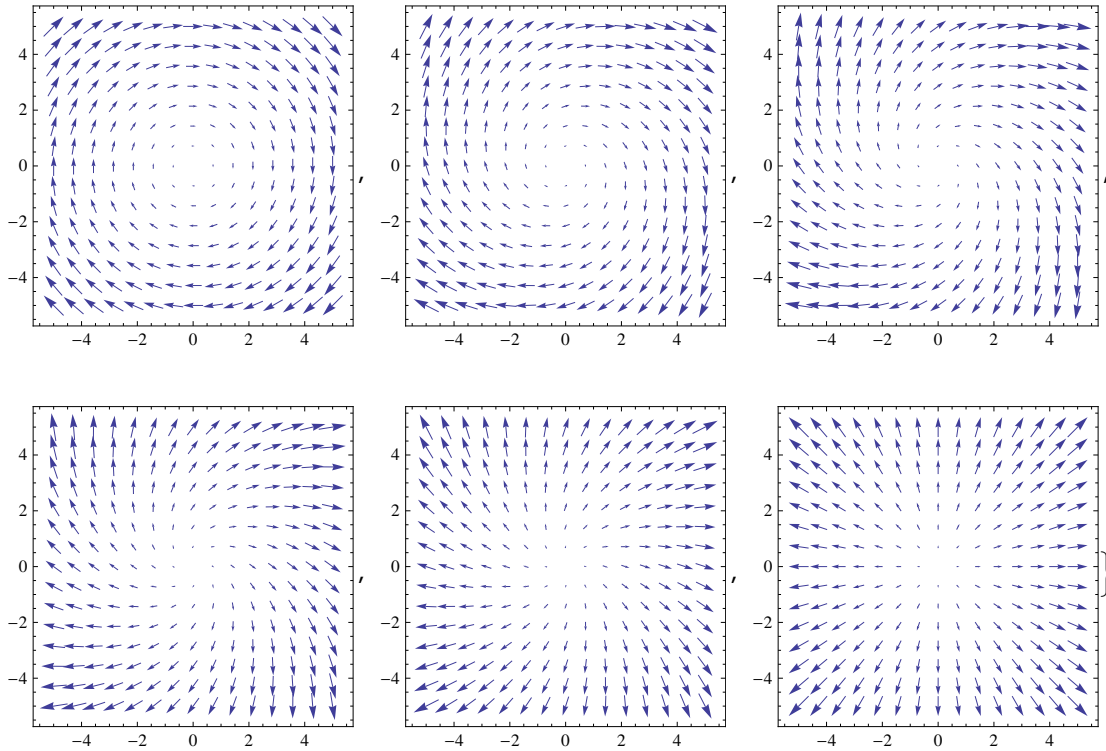
In[44]:=  $\mathbf{F} = \{x \cos[\alpha] - y \sin[\alpha], x \sin[\alpha] + y \cos[\alpha]\}$

Out[44]:=  $\{x \cos[\alpha] - y \sin[\alpha], y \cos[\alpha] + x \sin[\alpha]\}$

In[46]:= Table[VectorPlot[F, {x, -5, 5}, {y, -5, 5}], {alpha, 0, 2 pi, pi/10}]







In[102]:= (\* for the  
 checkers: it's impossible to draw this field because of the parameter  
 $\alpha$ . here we drew the field only for some selected values of  $\alpha$ . a  
 clear explanation in words with 1 picture is good enough \*)

In[53]:= (\* 1 \*)

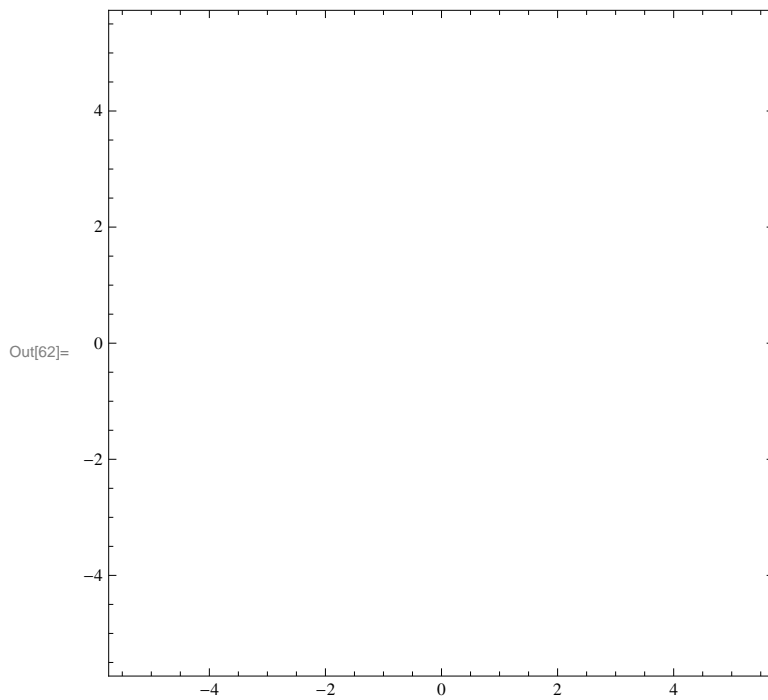
In[49]:=  $\mathbf{u} = 5$

Out[49]= 5

In[50]:=  $\nabla \mathbf{u} = \mathcal{D}[\mathbf{u}, \{\{x, y\}\}]$

Out[50]= {0, 0}

```
VectorPlot[Evaluate[ $\nabla u$ ], {x, -5, 5}, {y, -5, 5}]
```



```
In[100]:= (* zero vectors everywhere. a picture  
with points everywhere is also acceptable *)
```

```
In[54]:= (* 2 *)
```

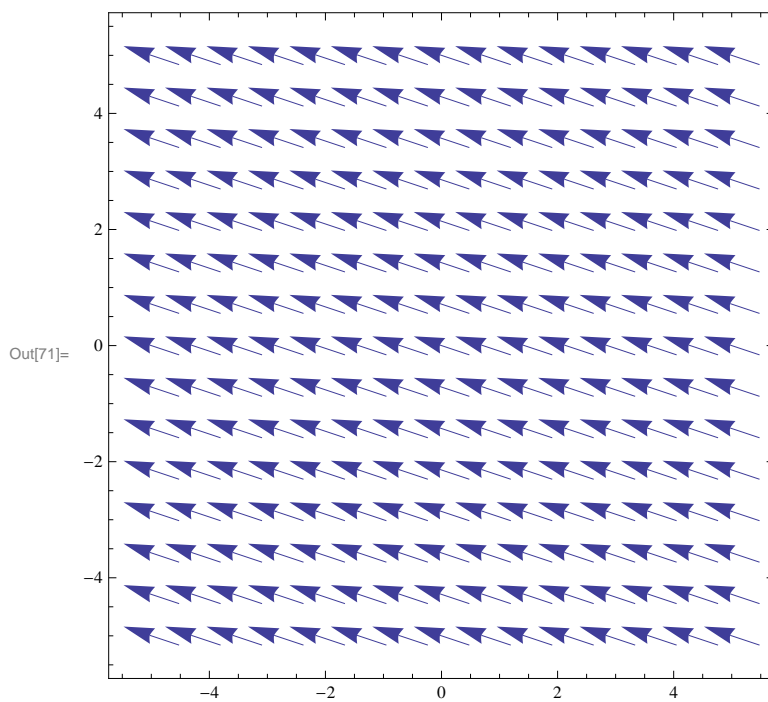
```
In[55]:=  $u = -3x + y$ 
```

```
Out[55]=  $-3x + y$ 
```

```
In[69]:=  $\nabla u = D[u, \{\{x, y\}\}]$ 
```

```
Out[69]=  $\{-3, 1\}$ 
```

```
In[71]:= VectorPlot[Evaluate[ $\nabla u$ ], {x, -5, 5}, {y, -5, 5}]
```





In[72]:= (**\* 3 \***)

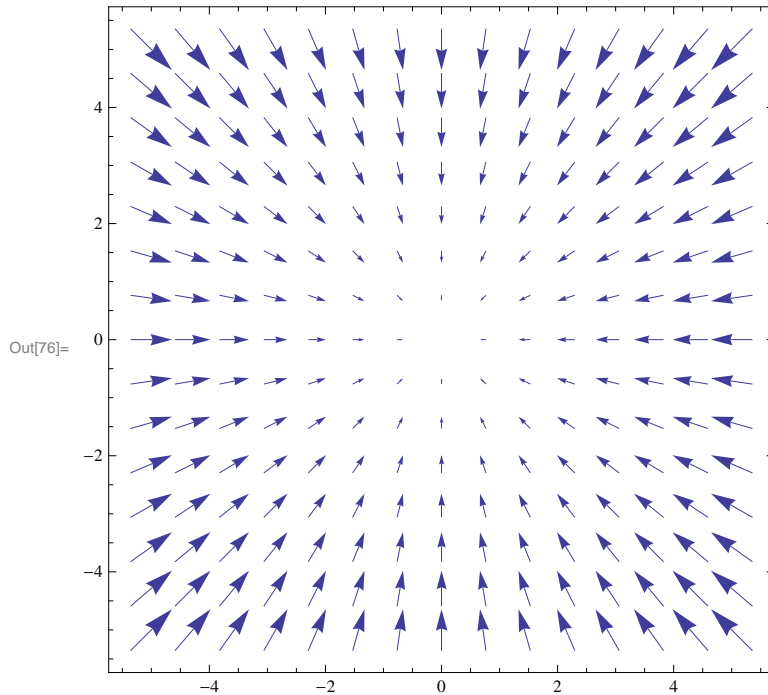
In[73]:=  $\mathbf{u} = -\mathbf{x}^2 - \mathbf{y}^2$

Out[73]=  $-\mathbf{x}^2 - \mathbf{y}^2$

In[74]:=  $\nabla \mathbf{u} = \mathbf{D}[\mathbf{u}, \{\{\mathbf{x}, \mathbf{y}\}\}]$

Out[74]=  $\{-2\mathbf{x}, -2\mathbf{y}\}$

In[76]:= `VectorPlot[Evaluate[ $\nabla \mathbf{u}$ ], { $\mathbf{x}$ , -5, 5}, { $\mathbf{y}$ , -5, 5}]`



In[77]:= (**\* 4 \***)

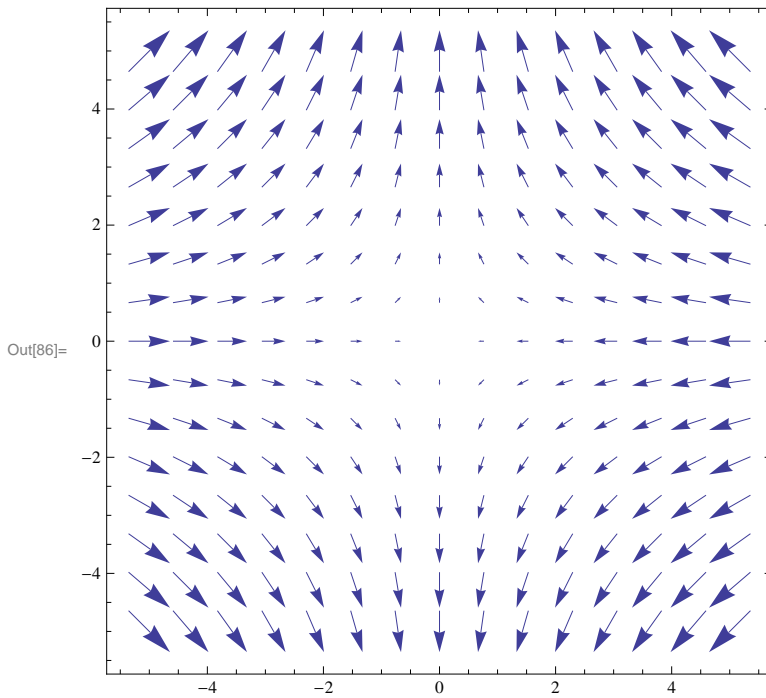
In[78]:=  $\mathbf{u} = -\frac{1}{2}\mathbf{x}^2 + \frac{1}{2}\mathbf{y}^2$

Out[78]=  $-\frac{\mathbf{x}^2}{2} + \frac{\mathbf{y}^2}{2}$

In[79]:=  $\nabla \mathbf{u} = \mathbf{D}[\mathbf{u}, \{\{\mathbf{x}, \mathbf{y}\}\}]$

Out[79]=  $\{-\mathbf{x}, \mathbf{y}\}$

In[86]:= `VectorPlot[Evaluate[ $\nabla u$ ], {x, -5, 5}, {y, -5, 5}]`



In[87]:= `(* 5 *)`

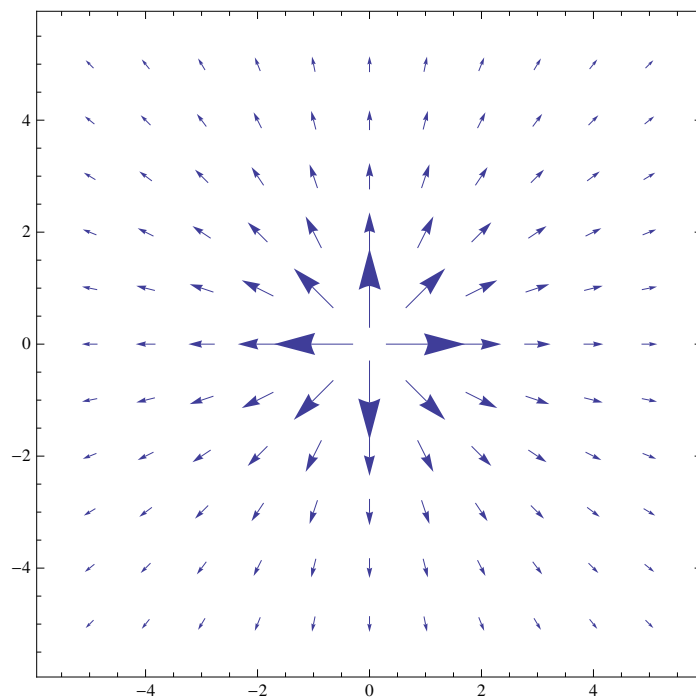
In[89]:= `u = Log[x2 + y2]`

Out[89]= `Log[x2 + y2]`

In[90]:=  `$\nabla u = D[u, {{x, y}}$ ]`

Out[90]= `{ $\frac{2x}{x^2 + y^2}, \frac{2y}{x^2 + y^2}$ }`

In[92]:= `VectorPlot[Evaluate[ $\nabla u$ ], {x, -5, 5}, {y, -5, 5}, VectorPoints -> {11, 11}]`



In[101]:= `(* 6 *)`

In[103]:=  $\mathbf{u} = \text{ArcTan}\left[\frac{y}{x}\right]$

Out[103]=  $\text{ArcTan}\left[\frac{y}{x}\right]$

In[104]:=  $\nabla \mathbf{u} = \mathbf{D}[\mathbf{u}, \{\{x, y\}\}]$

Out[104]=  $\left\{-\frac{y}{x^2 \left(1 + \frac{y^2}{x^2}\right)}, \frac{1}{x \left(1 + \frac{y^2}{x^2}\right)}\right\}$

In[105]:= **Simplify[%]**

Out[105]=  $\left\{-\frac{y}{x^2 + y^2}, \frac{x}{x^2 + y^2}\right\}$

In[110]:= **VectorPlot[Evaluate[ $\nabla \mathbf{u}$ ], {x, -5, 5}, {y, 0, 10}, VectorPoints  $\rightarrow$  {10, 10}]**

