1 Find a characterisation in real terms of the line through the points of intersection of two circles in the case that both these points are complex. Prove that it is the locus of points having the same power with respect to both circles. (The power of a point with respect to a circle is the square of the distance between it and the points of tangency of the tangent lines to the circle.)

2 Which rational functions $p(x)/q(x)$ are regular at the point at infinity of $\mathbb{P}^1$? What order of zero do they have there?

3 Prove that an irreducible cubic curve has at most one singular point, and that the multiplicity of a singular point is 2. If the singularity is a node then the cubic is projectively equivalent to the curve in (1.2); and if a cusp then to the curve $y^2 = x^3$.

4 What is the maximum multiplicity of intersection of two nonsingular conics at a common point?

5 Prove that if the ground field has characteristic p then every line through the origin is a tangent line to the curve $y = x^{p+1}$. Prove that over a field of characteristic 0, there are at most a finite number of lines through a given point tangent to a given irreducible curve.

6 Prove that the sum of multiplicities of two singular points of an irreducible curve of degree n is at most n, and the sum of multiplicities of any 5 points is at most $2n$.

7 Prove that for any two distinct points of an irreducible curve there exists a rational function that is regular at both, and takes the value 0 at one and 1 at the other.

8 Prove that for any nonsingular points $P_1, \dots, P_r$ of an irreducible curve and numbers $m_1, \dots, m_r \geq 0$ there exists a rational function that is regular at all these points, and has a zero of multiplicity m_i at P_i.

9 For what values of m is the cubic $x_0^3 + x_1^3 + x_2^3 + m x_0 x_1 x_2 = 0$ in $\mathbb{P}^2$ nonsingular? Find its inflexion points.

10 Find all the automorphisms of the curve of (1.2).

11 Prove that on the projective line and on a conic of $\mathbb{P}^2$, a rational function that is regular at every point is a constant.

12 Give an interpretation of Pascal's theorem in the case that pairs of vertexes of the hexagon coincide, and the lines joining them become tangents.