פתרון תרגיל בית מספר 3

אנליזה 2 תשע"ט

1. $\int \underbrace{x}_{V'}\underbrace{e^x}_{U}\, dx = xe^x - \int 1 \cdot e^x\, dx = xe^x - e^x + c$

2. $\int \underbrace{x^4}_{V'}\underbrace{lnx}_{U}\, dx = lnx \cdot \frac{x^5}{5} - \int \frac{1}{x} \cdot \frac{x^5}{5} dx = lnx \cdot \frac{x^5}{5} - \frac{1}{5}\int x^4\, dx = lnx \cdot \frac{x^5}{5} - \frac{1}{5} \cdot \frac{x^5}{5} + c =$

$\frac{x^5}{5}\left[lnx + \frac{1}{5}\right] + c$

3. $\int \underbrace{x}_{U}\underbrace{sinsx}_{V'}\, dx = x(-cosx) - \int 1(-cosx)\, dx = -xcosx + \int cosx\, dx =$

4. $\int \cos(lnx)\, dx = \begin{bmatrix} lnx = t \\ \frac{1}{x} dx = dt \\ dx = xdt \end{bmatrix} = \int \cos t\, xdt = [x = e^t] = \overbrace{\int \underbrace{\cos t}_{V'} \cdot \underbrace{e^t}_{U} dt}^{I} =$

$sint \cdot e^t - \int \underbrace{sint}_{V'} \cdot \underbrace{e^t}_{U} dt = sint \cdot e^t - (-cost)e^t + \int (-cost) \cdot e^t dt =$

$sint \cdot e^t + coste^t - \underbrace{\int cost \cdot e^t dt}_{I}$

$I = sint \cdot e^t + coste^t - I$

$2\, I = sint \cdot e^t + coste^t$

$I = \frac{1}{2}(\, sint \cdot e^t + coste^t) + c$

5. $\int \underbrace{e^{2x}}_{U}\underbrace{sin4x}_{V'}\, dx = e^{2x} \cdot \left(-\frac{1}{4}cos4x\right) - \int 2e^{2x} \cdot (-\frac{1}{4}cos4x)dx =$

$-\frac{1}{4}e^{2x} \cdot cos4x + \frac{1}{2}\int \underbrace{e^{2x}}_{U}\underbrace{cos4x}_{V'}\, dx =$

$-\frac{1}{4}e^{2x} \cdot cos4x + \frac{1}{2}\left[\frac{1}{4}e^{2x}sin4x - \frac{1}{2}\int e^{2x}sin4x\, dx\right] =$

$-\frac{1}{4}e^{2x} \cdot cos4x + \frac{1}{8}e^{2x}sin4x - \frac{1}{4}\overbrace{\int e^{2x}sin4x\, dx}^{I} = \overbrace{\int e^{2x}sin4x\, dx}^{I}$

$I = -\frac{1}{4}e^{2x} \cdot cos4x + \frac{1}{8}e^{2x}sin4x - \frac{1}{4}I$

$\frac{5}{4}I = -\frac{1}{4}e^{2x} \cdot cos4x + \frac{1}{8}e^{2x}sin4x$

$I = -\frac{1}{5}e^{2x} \cdot cos4x + \frac{1}{10}e^{2x}sin4x + c = \frac{1}{5}e^{2x}\left(-cos4x + \frac{1}{2}sin4x\right) + c$

$$\int \frac{1}{x^2-1}\,dx = \int \frac{1}{(x+1)(x-1)}\,dx$$ (א)

נפרק

$$\frac{1}{(x+1)(x-1)} = \frac{A}{x+1} + \frac{B}{x-1} = \frac{A(x-1)+B(x+1)}{(x+1)(x-1)}$$

נשווה מקדמים

$$1 = x(A+B) + B - A$$

$$\Rightarrow \left.\begin{array}{l} A+B=0 \\ B-A=1 \end{array}\right\} \Rightarrow \begin{array}{l} B=\frac{1}{2} \\ A=-\frac{1}{2} \end{array}$$

נחזור לאינטגרל

$$\int \frac{1}{x^2-1}\,dx = \int \frac{-\frac{1}{2}}{x+1} + \frac{\frac{1}{2}}{x-1}\,dx = -\frac{1}{2}\int \frac{1}{x+1}\,dx + \frac{1}{2}\int \frac{1}{x-1}\,dx$$

$$= -\frac{1}{2}\ln|x+1| + \frac{1}{2}\ln|x-1| + C$$

(ב) $$\int \frac{x}{x^2-4}\,dx = \frac{1}{2}\int \frac{2x}{x^2-4}\,dx = \frac{1}{2}\ln|x^2-4| + C$$

(ג) $$\int \frac{1}{x^2+4}\,dx = \int \frac{1}{x^2+2^2} = \frac{1}{2}\arctan\left(\frac{x}{2}\right) + C$$

(ב) $\int \frac{4}{x-1}\,dx = 4\int \frac{1}{x-1}\,dx = 4\ln|x-1| + C$

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(1) $\int \frac{3x+2}{x-1}\,dx = \int \frac{3x}{x-1}\,dx + \int \frac{2}{x-1}\,dx$

$= \int \frac{3x-3+3}{x-1}\,dx + \int \frac{2}{x-1}\,dx$

$= \int 3 + \frac{3}{x-1}\,dx + \int \frac{2}{x-1}\,dx = 3x + 3\ln|x-1|$

$+ 2\ln|x-1| + C$

(ג) $\int \frac{2x+1}{7x+1}\,dx = \int \frac{2x}{7x+1}\,dx + \underbrace{\int \frac{1}{7x+1}\,dx}$

$= \underbrace{\frac{2}{7}\int \frac{7x}{7x+1}\,dx}_{(*)} + \underbrace{\frac{1}{7}\int \frac{7}{7x+1}\,dx}_{\frac{1}{7}\ln|7x+1|} =$

$\int \frac{7x}{7x+1}\,dx = \int \frac{7x+1-1}{7x+1}\,dx = \int 1 - \frac{1}{7x+1}\,d$ (*) קוח

$= x - \frac{1}{7}\ln|7x+1|$

קוח

$$\int \frac{2x+1}{7x+1} = \frac{2}{7}\left(x - \frac{1}{7}\ln|7x+1|\right) + \frac{1}{7}\ln|7x+1| + C$$

$$= \frac{2}{7}x - \frac{2}{49}\ln|7x+1| + \frac{1}{7}\ln|7x+1| + C$$

$$= \frac{2}{7}x + \frac{5}{49}\ln|7x+1| + C$$

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$$= \ln|x+2| + C$$