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 $A \in F^{n \times n}$   $f_A(x)$ -  
 $f_A(x) = (f_1(x))^{r_1} (f_2(x))^{r_2} \dots (f_k(x))^{r_k}$  :  
 : , ( )  
 $m_A(x) = (f_1(x))^{s_1} (f_2(x))^{s_2} \dots (f_k(x))^{s_k}$   
 $s_1, \dots, s_k$   
 : ( )  $1 \leq i \leq k$   
 $\text{rank}(f_i(A)), \text{rank}(f_i(A))^2, \text{rank}(f_i(A))^3 \dots$   
 . ,  
 $s_i$

$$A = \begin{pmatrix} 0 & 4 & 1 & -2 \\ -1 & 4 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ -1 & 3 & 0 & 0 \end{pmatrix}$$

$$f_A(x) = (x-1)^3(x-2)$$

$$: s_1 \quad m_A(x) = (x-1)^{s_1}(x-2)$$

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$$\text{rank}(A-I) = \text{rank} \begin{pmatrix} -1 & 4 & 1 & -2 \\ -1 & 3 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ -1 & 3 & 0 & -1 \end{pmatrix} = 2$$

$$\text{rank}(A-I)^2 = \text{rank} \begin{pmatrix} -1 & 2 & -1 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 \end{pmatrix} = 1$$

$$\text{rank}(A-I)^3 = \text{rank} \begin{pmatrix} -1 & 2 & -1 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 \end{pmatrix} = 1$$

$$m_A(x) = (x-1)^2(x-2) \cdot s_1 = 2$$

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$\{v_1, \dots, v_n\}$  .  $A \in F^{n \times n}$

$\{v_i, Av_i, A^2v_i, \dots\} : 1 \leq i \leq n \quad . F^n$

. "  $\{v_i, Av_i, A^2v_i, \dots, A^d v_i\} \quad d$

"  $\{v_i, Av_i, A^2v_i, \dots, A^d v_i, A^{d+1}v_i\}$

"  $r_0v_i + r_1Av_i + r_2A^2v_i + \dots + r_dA^d v_i + r_{d+1}A^{d+1}v_i = 0$

$$. m_i(x) = r_0 + r_1x + r_2x^2 + \dots + r_dx^d + r_{d+1}x^{d+1}$$

.  $\{m_1, \dots, m_n\}$  (lcm)  $A$

$$) A = \begin{pmatrix} 2 & 3 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \end{pmatrix} \in \mathbb{R}^{4 \times 4}$$

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.  $\{e_1, \dots, e_4\} :$

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$$A^2 = \begin{pmatrix} 4 & 12 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 9 \end{pmatrix}$$

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$$m_1(x) = x - 2 : Ae_1 - 2e_1 = 0 \quad \{e_1, Ae_1\} = \{(1,0,0,0), (2,0,0,0)\}$$

$$\{e_2, Ae_2, A^2e_2\} = \{(0,1,0,0), (3,2,0,0), (12,4,0,0)\}$$

$$m_2(x) = x^2 - 4x + 4 = (x - 2)^2 : A^2e_2 - 4Ae_2 + 4e_2 = 0$$

$$m_3(x) = x - 2 \quad \{e_3, Ae_3\} = \{(0,0,1,0), (0,0,2,0)\}$$

$$m_4(x) = x - 3 \quad \{e_4, Ae_4\} = \{(0,0,0,1), (0,0,0,3)\}$$

$$lcm(m_1, m_2, m_3, m_4) = (x - 2)^2 (x - 3)$$

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$$f_A(x) = f_1^{r_1} \cdots f_k^{r_k} :$$

$$m_A(A) = 0$$

$$m_A(x) = f_1 \cdots f_k$$

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$$A = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

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$$f_A(x) = (x-1)^2(x-2) :$$

$$(x-1)(x-2)$$

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$$.m_A(x) = (x-1)(x-2)$$

$$m_A(A) = (A-I)(A-2I) = A^2 - 3A + 2I = 0$$

$$\Rightarrow A(A-3I) = -2I \Rightarrow A\left(\frac{3}{2}I - \frac{1}{2}A\right) = I \Rightarrow A^{-1} = \frac{3}{2}I - \frac{1}{2}A$$

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