א)

$$\int x\ln^2 x\,dx$$

פתרון:

$u=\ln^2 x \qquad dv=x$

$du=\frac{2\ln x}{x} \qquad v=\frac{x^2}{2}$

$$\int x\ln^2 x\,dx=\frac{x^2\ln^2 x}{2}-\int x\ln x\,dx=$$

$u=\ln x \qquad dv=x$

$du=\frac{1}{x}dx \qquad v=\frac{x^2}{2}$

$$=\frac{x^2\ln^2 x}{2}-\left[\frac{x^2\ln x}{2}-\frac{1}{2}\int x\,dx\right]=$$

$$=\frac{x^2\ln^2 x}{2}-\frac{x^2\ln x}{2}+\frac{x^2}{2}+c$$

ב)

$$\int \tan(x)\,dx$$

פתרון:

$\cancel{u=\tan x \qquad dv=dx}$

$\cancel{du=\frac{dx}{\cos^2 x} \qquad v=x}$

$$\int \tan x\,dx=\int\frac{\sin x}{\cos x}dx=\int\frac{1}{t}(-dt)=$$

$t=\cos x$

$dt=-\sin x\,dx$

$$=-\ln|t|+c=$$

$$=-\ln|\cos x|+c$$

ג)

$$\int\frac{dx}{\sqrt{x}(x+1)}=$$

פתרון:

$t^2=x \qquad dx=2t\,dt$

$$=\int\frac{2\cancel{t}\,dt}{\cancel{t}(t^2+1)}=2\arctan t+c=2\arctan\sqrt{x}+c$$

ג.

$$\int x^k \ln x\,dx = \frac{x^{k+1}\ln x}{k+1} - \int \frac{x^{k+1}\,dx}{x(k+1)} =$$

$$u = \ln x \quad dv = x^k\,dx$$
$$du = \frac{dx}{x} \quad v = \frac{x^{k+1}}{k+1}$$

$$= \frac{x^{k+1}\ln x}{k+1} - \int \frac{x^k\,dx}{k+1} =$$

$$= \frac{x^{k+1}\ln x}{k+1} - \frac{x^{k+1}}{(k+1)^2} + c$$

ד. לשון כללי.

ה.

$$\int 2x \arctan x\,dx = 2\left[\frac{x^2 \arctan}{2} - \frac{1}{2}\int \frac{x^2}{1+x^2}\right]$$

$$u = \arctan x \quad dv = x$$
$$du = \frac{dx}{1+x^2} \quad v = \frac{x^2}{2}$$

$$\int \frac{x^2}{1+x^2} = \int \frac{x^2+1-1}{x^2+1} = \int dx - \int \frac{dx}{x^2+1} =$$

$$= x - \arctan x + c$$

סה״כ נקבל:

$$x^2 \arctan x - x + \arctan x + c$$

ו.

$$\int x \arctan(x^2)\,dx = \frac{1}{2}\int \arctan(x^2)\cdot 2x\,dx =$$

$$t = x^2$$
$$dt = 2x\,dx$$

$$= \frac{1}{2}\int \arctan(t)\,dt = \frac{1}{2}\left[t\arctan(t) - \int \frac{t}{t^2+1}\,dt\right] =$$

$$u = \arctan t \quad dv = dt$$
$$du = \frac{1}{t^2+1}\,dt \quad v = t$$

$$= \frac{1}{2}\left[t\arctan t - \int \frac{du}{2\cdot u}\right] =$$

$$= \frac{1}{2}\left[t\arctan t - \frac{1}{2}\ln(t^2+1)\right] + c =$$

$$= \frac{1}{2}\left[x^2\arctan x^2 - \frac{1}{2}\ln(x^4+1)\right] + c$$

ה)

$$\int x^2 \sin(2x)\,dx$$

$$t = 2x \qquad dt = 2dx \Rightarrow$$

$$\Rightarrow \int \frac{t^2}{4} \sin(t)\,dt = \frac{1}{4}\int t^2 \sin(t)\,dt =$$

$$\boxed{\begin{array}{ll} u = t^2 & dv = \sin t \\ du = 2t & v = -\cos(t) \end{array}} \quad = \frac{1}{4}\left[-t^2 \cos t + \int 2t\cos(t)\right] =$$

$$= \frac{1}{4}\left[-t^2 \cos t + 2\left[t\sin t - \int \sin t\,dt\right]\right] =$$

$$\boxed{\begin{array}{ll} u = t & dv = \cos t \\ du = dt & v = \sin t \end{array}} \quad = \frac{1}{4}\left[-t^2 \cos t + 2\left[t\sin t + \cos t\right]\right] =$$

$$= \frac{1}{4}\left[-4x^2 \cos 2x + 2\left[2x \sin 2x + \cos 2x\right]\right] =$$

$$= -x^2 \cos 2x + x \sin 2x + \frac{\cos 2x}{2} + C$$

ו)

$$\int (\sin(x))^{2k+1} (\cos(x))^{2k}$$

<u>פתרון:</u>

<u>שיטה א':</u> נשתמש בטבלה של כיתה:

כאן החזקה של sin היא אי זוגית ולכן נשתמש

הצבה: $t = \cos x \qquad dt = -\sin x\,dx$

$$\sin x = \sqrt{1-t^2} \qquad dx = \frac{-dt}{\sin x}$$

<u>לכן:</u>

$$\int \left(\sqrt{1-t^2}\right)^{2k+1} \cdot t^{2k} \left(-\frac{dt}{\sqrt{1-t^2}}\right) = \int \left(\sqrt{1-t^2}\right)^{2k} \cdot \left(-t^{2k}\right) dt =$$

$$= \int (1-t^2)^k (-t)^{2k}\,dt = \ldots$$

<u>שיטה ב':</u> נוסחה

$$S_n = \int (\sin x)^n\,dx$$

$$S_n = \int (\sin x)^{n-1}(1-\cos^2 x)\,dx = S_{n-2} - \int (\sin x)^{n-2} \cos x \cos x\,dx =$$

$$= S_{n-2} - \frac{(\sin x)^{n-1}\cos x}{n-1} - \frac{1}{n-1} S_n$$

$$S_n = -\frac{(\sin x)^{n-1}\cos x}{n} + \frac{n-1}{n} S_{n-2}$$

[illegible]

$$\int (\sin x)^{2k+1} (\cos x)^{2k}\, dx = \int (\sin x)^m (1-\sin^2 x)^k =$$

$2k+1=m$

$$= S_m - k\, S_{m+2} + \dots + (-1)^k S_{m+2k}$$

[illegible]

$$\int \sin^2 x \cos^2 x = 4 \int (2\sin 2x)^2\, dx =$$

$$= 4 \int [1 - \cos 4x]\, dx = 4\left[x - \frac{\sin 4x}{4}\right] + c =$$

$$= 4x - \sin 4x + c$$