CHAPTER FIVE

The Logic of Relations

I. SYMBOLIZING RELATIONS

Some propositions which contain two or more proper names (of individuals) are correctly interpreted as truth-functional compounds of singular propositions having different subject terms. For example, the proposition

Lincoln and Grant were presidents.

is properly interpreted as the conjunction of the two singular propositions

Lincoln was a president   and   Grant was a president.

But for some other propositions having the same verbal pattern that analysis is wholly unsatisfactory. Thus the proposition

Lincoln and Grant were acquainted.

is definitely *not* a conjunction or any other truth function of the two expressions

Lincoln was acquainted   and   Grant was acquainted.

On the contrary, dividing the proposition in this way destroys its significance, for its meaning is not that both Lincoln and Grant were (or had) acquaintances, but that they were *acquainted with each other*. The given proposition does not assert that Lincoln and Grant both had a certain *property*, but that they stood in a certain *relationship*. Lincoln is not said simply to be acquainted (whatever that might mean), but *acquainted with Grant*. Other propositions which express relations between two individuals are

John loves Mary.
Plato was a student of Socrates.
Isaac was a son of Abraham.
New York is east of Chicago.
Chicago is smaller than New York.

Relations such as these, which can hold between two individuals, are called 'binary' or 'dyadic'. Other relations may relate three or more individuals. For example, the propositions

Detroit is between New York and Chicago.
Helen introduced John to Mary.
America won the Phillipines from Spain.

express *ternary* or *triadic* relations, while *quaternary* or *tetradic* relations are expressed by the propositions

America bought Alaska from Russia for seven million dollars.
Jack traded his cow to the peddler for a handful of beans.
Al, Bill, Charlie, and Doug played bridge together.

Relations enter into arguments in various ways. One example of a relational argument is

Al is older than Bill.
Bill is older than Charlie.
Therefore, Al is older than Charlie.

A slightly more complex example, which involves quantification, is this:

> Helen likes David.
> Whoever likes David likes Tom.
> Helen likes only good-looking men.
> ———
> Therefore, Tom is a good-looking man.

A still more complex relational inference, which involves multiple quantification, is:

> All horses are animals.
> ———
> Therefore, the head of a horse is the head of an animal.

The latter is a valid inference which, as De Morgan observed, all the logic of Aristotle will not permit one to draw. Its validation by our apparatus of quantifiers and propositional functions will be set forth in the next section.

Before discussing the validation of relational arguments, which will require no methods of proof beyond those developed in the preceding chapter, the problem of *symbolizing* relational propositions must be dealt with. Just as a single predicate symbol can occur in different propositions, so a single relation symbol can occur in different propositions. Just as we have the predicate 'human' common to the propositions:

> Aristotle is human.
> Plato is human.
> Socrates is human.

so we have the relational word 'teacher' common to the propositions:

> Socrates was a teacher of Plato.
> Plato was a teacher of Aristotle.

And just as we regard the three subject-predicate propositions as different substitution instances of the propositional function '*x* is human', so we can regard the two relational propositions as different substitution instances of the propositional function '*x* was a teacher of *y*'. Replacing the variable '*x*' by the constant 'Socrates' and the variable '*y*' by the constant 'Plato' gives us the first proposition; replacing the '*x*' by 'Plato' and the '*y*' by 'Aristotle' gives the second. The *order* of replacement is of great importance here: if '*x*' is replaced by 'Aristotle' and '*y*' by 'Plato', the result is the *false* proposition

> Aristotle was a teacher of Plato.

Just as a propositional function of one variable like '*x* is human' was abbreviated as '*Hx*', so a propositional function of two variables like '*x* was the teacher of *y*' is abbreviated as '*Txy*'. Similarly, the propositional function '*x* is between *y* and *z*' will be abbreviated as '*Bxyz*', and the propositional function '*x* traded *y* to *z* for *w*' will be abbreviated as '*Txyzw*'. Our first specimen of a relational argument, since it involves no quantifications, is very easily symbolized. Using the individual constants '*a*', '*b*', and '*c*' to denote Al, Bill, and Charlie, and the expression '*Oxy*' to abbreviate '*x* is older than *y*', we have

$$Oab$$
$$Obc$$
$$\therefore Oac$$

Our second argument is not much more difficult, since none of its propositions contains more than a single quantification. Using the individual constants '*h*', '*d*', and '*t*' to denote Helen, David, and Tom, respectively, '*Gx*' to abbreviate '*x* is a good-looking man', and the symbol '*Lxy*' to abbreviate '*x* likes *y*', the argument can be symbolized as

1. Lhd
2. $(x)(Lxd \supset Lxt)$
3. $(x)(Lhx \supset Gx)$

$\therefore Gt$

The demonstration of its validity is so easily constructed that it may well be set down now, before going on to consider some of the more difficult problems of symbolization. Referring back to the numbered premisses above, the demonstration proceeds:

4. $Lhd \supset Lht$	2, **UI**
5. Lht	4, 1, M.P.
6. $Lht \supset Gt$	3, **UI**
7. Gt	6, 5, M.P.

Symbolizing relational propositions becomes more complicated when several quantifications occur in a single proposition. Our discussion of the problem will be simplified by confining attention at first to two individual constants, '*a*', and '*b*', and the propositional function '*x* attracts *y*', abbreviated as '*Axy*'. The two statements '*a* attracts *b*' and '*b* is attracted by *a*' obviously have the same meaning, the first expressing that meaning by use of the *active voice*, the second by use of the *passive voice*. Both statements translate directly into the single formula '*Aab*'. Similarly, the two statements '*b* attracts *a*' and '*a* is attracted by *b*' are both symbolized by the formula '*Aba*'. These two different substitution instances of '*Axy*' are logically independent of each other, either can be true without entailing the truth of the other.

We are still on elementary and familiar ground when we come to symbolize

'*a* attracts everything'
'everything is attracted by *a*' } as '$(x)Aax$',
'a attracts something'
'something is attracted by *a*' } as '$(\exists x)Aax$',
'everything attracts *a*'
'*a* is attracted by everything' } as '$(x)Axa$',
'something attracts *a*'
'*a* is attracted by something' } as '$(\exists x)Axa$'.

But the problem of symbolizing becomes more complex when we dispense entirely with individual constants and consider relational propositions which are completely general. The simplest propositions of this kind are

1. Everything attracts everything.
2. Everything is attracted by everything.
3. Something attracts something.
4. Something is attracted by something.
5. Nothing attracts anything.
6. Nothing is attracted by anything.

which are symbolized by the following formulas:

1. $(x)(y)Axy$
2. $(y)(x)Axy$
3. $(\exists x)(\exists y)Axy$
4. $(\exists y)(\exists x)Axy$
5. $(x)(y) \sim Axy$
6. $(y)(x) \sim Axy$

In their English formulations, propositions 1 and 2 are clearly equivalent to each other, as are 3 and 4, and 5 and 6. The first two equivalences are easily established for the corresponding logical formulas:

→1. $(x)(y)Axy$		→1. $(\exists x)(\exists y)Axy$	
2. $(y)Awy$	1, **UI**	2. $(\exists y)Awy$	1, **EI**
3. Awv	2, **UI**	3. Awv	2, **EI**
4. $(x)Axv$	3, **UG**	4. $(\exists x)Axv$	3, **EG**
5. $(y)(x)Axy$	4, **UG**	5. $(\exists y)(\exists x)Axy$	4, **EG**
6. $(x)(y)Axy \supset (y)(x)Axy$	1–5, C.P.	6. $(\exists x)(\exists y)Axy \supset (\exists y)(\exists x)Axy$	1–5, C.P.

These demonstrate the logical truth of conditionals rather than of equivalences, but that their converses are true also can be established by simply reversing the orders of steps 1 through 5. (The equivalence between formulas 5 and 6 is clearly established by the same pattern of argument that proves 1 equivalent to 2.)

When we turn to the next pair of statements

7. Everything attracts something.
8. Something is attracted by everything.

there is no longer any logical equivalence or sameness of meaning. Sentence 7 is not entirely unambiguous, and some exceptional contexts might shift its meaning, but its most natural interpretation is *not* that there is some one thing which is attracted by everything, but rather that everything attracts *something or other*. We can approach its symbolization by way of successive paraphrasings, writing first

(x)(x attracts something)

and then symbolizing the expression 'x attracts something' the same way in which we symbolized 'a attracts something'. This gives us the formula

7. $(x)(\exists y)Axy$.

Sentence 8 is also susceptible of alternative interpretations, one of which would make it synonymous with sentence 7, meaning that something or other is attracted by any (given) thing. But a perfectly straightforward way of understanding sentence 8 is to take it as asserting that some *one thing* is attracted by all things. Its symbolization, too, can be accomplished in a stepwise fashion, writing first

$(\exists y)$(y is attracted by everything)

and then symbolizing the expression 'y is attracted by everything' the same way in which we symbolized 'a is attracted by everything'. This gives us the formula

8. $(\exists y)(x)Axy$.

There is a certain *misleading* similarity between formulas 7 and 8. They both consist of the propositional function 'Axy' to which are applied a universal quantifier with respect to 'x' and an existential quantifier with respect to 'y'. But the *order* in which the quantifiers are written is different in each case, and that makes a world of difference in their meanings. Formula 7, in which the universal quantifier comes first, asserts that given anything in the universe, there is something or other which it attracts. But formula 8, in which the existential quantifier comes first, asserts that there is some one thing in the universe such that everything in the universe attracts *it*. Where two quantifiers are applied to one propositional function, if they are both universal or both existential, their order does not matter, as is shown by the equivalence of formulas 1 and 2, 3 and 4, and 5 and 6. But where one is universal and the other existential the order of generalization or quantification is very important indeed.

Although formulas 7 and 8 are not equivalent, they are not independent. The former is validly deducible from the latter. The demonstration is easily constructed as follows:

1.	$(\exists y)(x)Axy$	
2.	$(x)Axv$	1, **EI**
3.	Auv	2, **UI**
4.	$(\exists y)Auy$	3, **EG**
5.	$(x)(\exists y)Axy$	4, **UG**

But the inference is valid only one way. Any attempt to derive formula 8 from 7 must inevitably run afoul of one of the restrictions on **UG**.

A similar pair of inequivalent propositions may be written as

9. Everything is attracted by something.
10. Something attracts everything.

These are clearly inequivalent when the 'something' in 9, coming at the end, is understood as 'something or other', and the 'something' in 10, coming at the beginning, is understood as 'some one thing'. They are symbolized as

9. $(y)(\exists x)Axy$.
10. $(\exists x)(y)Axy$.

Relational propositions are sometimes formulated as though they were simple subject-predicate assertions. For example, 'a was struck' is most plausibly interpreted to assert that *something struck a*. Such implicit occurrences of relations are often marked by the passive voice of a transitive verb. Our symbolization of propositions containing implicit relations should be guided by consideration of the use to which they are to be put. Our motive in symbolizing arguments is to get them into that form which is most convenient for testing their validity by the application of our rules. Our goal, therefore, with respect to a given argument, is not that of providing a theoretically complete analysis, but rather of providing one sufficiently complete for the purpose at hand—the testing of validity. Consequently some implicit relations may be left implicit, while others require a more thorough analysis, as may be made clear by an example. Consider the argument

> Whoever visited the building was observed. Anyone who had observed Andrews would have remembered him. Nobody remembered Andrews. Therefore, Andrews didn't visit the building.

The first proposition of this argument contains two relations, one explicit, the other implicit. Explicitly, we have the relation of *someone visiting the building*. It is explicit because mention is made both of the visitor and what was visited by him. Implicitly, we have the relation of *someone observing someone*, which is implicit because no mention is made of the someone who does the observing—the omission being marked by the use of the passive voice. However, because the only other occurrence of 'x visited the building' is also as a *unit*, in the conclusion, it need not be treated as a relation at all, but may be symbolized as a simple predicate. On the other hand, 'x observed y', despite its merely implicit occurrence in the first premiss, must be explicitly symbolized as a relation if the validity of the argument is to be proved. For its second occurrence is not a simple repetition of the original unit; it appears instead as an explicit relation, with the first variable quantified and the second replaced by the proper name 'Andrews'. Using 'a' to denote Andrews, 'Vx' to abbreviate 'x visited the building', 'Oxy' to abbreviate 'x observed y', and 'Rxy' to abbreviate 'x remembers y', a symbolic translation and validation of the given argument may be written as

1.	$(x)[Vx \supset (\exists y)Oyx]$	
2.	$(x)[Oxa \supset Rxa]$	
3.	$(x) \sim Rxa$	$/ \therefore \sim Va$
4.	$Oza \supset Rza$	2, UI
5.	$\sim Rza$	3, UI
6.	$\sim Oza$	4, 5, M.T.
7.	$(y) \sim Oya$	6, UG
8.	$\sim(\exists y)Oya$	7, QN
9.	$Va \supset (\exists y)Oya$	1, UI
10.	$\sim Va$	9, 8, M.T.

Our demonstration of the validity of this argument would not have been helped at all by symbolizing 'Andrews visited the building' as a substitution instance of the relational 'x visited y' rather than of the simpler 'Vx'. But our demonstration absolutely required us to symbolize 'was observed' explicitly as a relation.

While on the subject of implicit or concealed relations, mention must be made of the philosophically interesting but logically troublesome topic of *pseudo-relations*. Examples of these are *desiring*, *hoping*, *planning*, *wishing-for*, and the like. These can be regarded as *pseudo-relations* because of the fact that certain inferences which are valid in connection with ordinary relations break down or are invalid when made with respect to *apparent* relations of the sort mentioned. If I *attend* a picnic, there must exist a picnic for me to attend. But if I merely *plan* a picnic, and never execute my plans, there need not exist any picnic at all. If I marry a perfect wife, there must exist a perfect wife for me to marry. But if I merely *desire* a perfect wife, it by no means follows that there exists a perfect wife to whom I stand in the relation of desiring. The existence of Santa Claus is not established by believing in him, for *believing in* is a pseudo rather than a genuine relation. We must beware of imputing existence to non-existents by mistaking pseudo-relations for genuine ones.

Most of our previous examples were illustrations of *unlimited* generality, in which it was asserted that *everything* stood in such-and-such a relation, or something did, or nothing did. A great many relational propositions are not so sweeping. Most assertions are more modest, claiming not that *everything* stands in such-and-such a relation, but that everything does *if* it satisfies certain conditions or restrictions. Thus we may say either that

Everything is attracted by all magnets.

or that

Everything made of iron is attracted by all magnets.

The second, of course, is the more modest assertion, being less *general* than the first. While the first is adequately symbolized, where '*Mx*' abbreviates '*x* is a magnet', as

$$(x)(y)[My \supset Ayx],$$

the second is symbolized, where '*Ix*' abbreviates '*x* is made of iron', as

$$(x)[Ix \supset (y)(My \supset Ayx)].$$

That the symbolization is correct can be seen by paraphrasing the second proposition in English as

Given anything at all, *if* it is made of iron then it is attracted by all magnets.

Perhaps the best way to symbolize relational propositions is by the kind of stepwise process that has already been exemplified. Let us illustrate it further, this time for propositions of limited generality. First let us consider the proposition

Any good amateur can beat some professional.

As a first step we may write

$$(x)\{(x \text{ is a good amateur}) \supset (x \text{ can beat some professional})\}.$$

Next, the consequent of the conditional between the braces

x can beat some professional

is symbolized as a generalization or quantified expression:

$$(\exists y)[(y \text{ is a professional}) \cdot (x \text{ can beat } y)].$$

Now, using the obvious abbreviations, '*Gx*', '*Px*', and '*Bxy*' for '*x* is a good amateur', '*x* is a professional', and '*x* can beat *y*', the given proposition is symbolized by the formula

$$(x)[Gx \supset (\exists y)(Py \cdot Bxy)].$$

Using the same method of paraphrasing by successive steps, we may symbolize

Some professionals can beat all amateurs.

first as

$$(\exists x)[(x \text{ is a professional}) \cdot (x \text{ can beat all amateurs})]$$

then as

$$(\exists x)\{(x \text{ is a professional}) \cdot (y)[(y \text{ is an amateur}) \supset (x \text{ can beat } y)]\}$$

and finally (using abbreviations) as

$$(\exists x)[Px \cdot (y)(Ay \supset Bxy)].$$

The same method is applicable in more complex cases, where more than one relation is involved. We symbolize the proposition

Anyone who promises everything to everyone is certain to disappoint somebody.

first by paraphrasing it as

$$(x)\{[(x \text{ is a person}) \cdot (x \text{ promises everything to everyone})] \supset [x \text{ disappoints somebody}]\}.$$

The second conjunct of the antecedent

x promises everything to everyone

may be further paraphrased, first as

$$(y)[(y \text{ is a person}) \supset (x \text{ promises everything to } y)]$$

and then as

$$(y)[(y \text{ is a person}) \supset (z)(x \text{ promises } z \text{ to } y)].$$

The consequent in our first paraphrase

x disappoints somebody

has its structure made more explicit by being rewritten as

$$(\exists u)[(u \text{ is a person})\cdot(x \text{ disappoints } u)].$$

The original proposition can now be rewritten as

$$(x)\{\{(x \text{ is a person})\cdot(y)[(y \text{ is a person}) \supset (z)(x \text{ promises } z \text{ to } y)]\} \supset (\exists u)[(u \text{ is a person})\cdot(x \text{ disappoints } u)]\}.$$

Using the obvious abbreviations, '*Px*', '*Pxyz*', '*Dxy*' for '*x* is a person', '*x* promises *y* to *z*', and '*x* disappoints *y*', the proposition can be expressed more compactly in the formula

$$(x)\{\{Px\cdot(y)[Py \supset (z)Pxzy]\} \supset (\exists u)(Pu\cdot Dxu)\}.$$

With practice, of course, not all such intermediate steps need be written cut explicitly.

Quantification words such as 'everyone', 'anyone', 'everybody', 'anybody', and 'whoever', refer to *all persons* rather than to *all things;* and such quantification words as 'someone' and 'somebody' refer to *some persons* rather than to *some things*. It is frequently desirable to represent this reference in our symbolization. But doing so is not always necessary for the purpose of evaluating arguments containing these words, however, and the choice of symbolization procedure is determined on the same grounds on which one decides whether a relational clause or phrase is to be symbolized explicitly as a relation or as a mere predicate.

The words 'always', 'never', and 'sometimes' frequently have a strictly non-temporal significance, as in the propositions

> Good men always have friends.
> Bad men never have friends.
> Men who have no wives sometimes have friends.

which may be symbolized, using obvious abbreviations, as

$$(x)[(Gx\cdot Mx) \supset (\exists y)Fxy]$$
$$(x)[(Bx\cdot Mx) \supset \sim(\exists y)Fxy]$$
$$(\exists x)\{[Mx\cdot\sim(\exists y)(Wy\cdot Hxy)]\cdot(\exists z)Fxz\}.$$

However, some uses of these words are definitely temporal, and when they are, they can be symbolized by the logical machinery already available, as can other temporal words like 'while', 'when', 'whenever', and the like. An example or two should serve to make this clear. Thus the proposition

> Dick always writes Joan when they are separated.

asserts that all times when Dick and Joan are separated are times when Dick writes Joan. This can be symbolized using '*Tx*' for '*x* is a time', '*Wxyz*' for '*x* writes *y* at (time) *z*', and '*Sxyz*' for '*x* and *y* are separated at (time) *z*', as

$$(x)\{Tx \supset [Sdjx \supset Wdjx]\}$$

Perhaps the most vivid illustration of the adaptability of the present notation is in symbolizing the following remark, usually attributed to Lincoln:

> You can fool some of the people all of the time, and
> all of the people some of the time, but you cannot
> fool all of the people all of the time.

The first conjunct: 'You can fool some of the people all of the time' is ambiguous. It may be taken to mean either that *there is at least one person who can always be fooled* or that *for any time there is at least one person (or other) who can be fooled at that time.* Adopting the first interpretation, and using '*Px*' for '*x* is a person', '*Tx*' for '*x* is a time', and '*Fxy*' for 'you can fool *x* at (or during) *y*', the above may be symbolized as

$$\{(\exists x)[Px\cdot(y)(Ty \supset Fxy)]\cdot(\exists y)[Ty\cdot(x)(Px \supset Fxy)]\}\cdot(\exists y)(\exists x)[Ty\cdot Px\cdot\sim Fxy)].$$

The actual testing of relational arguments presents no new problems—once the translations into logical symbolism are effected. The latter is the more troublesome part, and so a number of exercises are provided for the student to do before going on.

EXERCISES

I. Using the following 'vocabulary', translate the given formulas into idiomatic English sentences:

Ax-*x* is silver	*Axy*-*x* helps *y*
Bx-*x* is blissful	*Bxy*-*x* belongs to *y*
Cx-*x* is a cloud	*Bxyz*-*x* borrows *y* from *z*
Dx-*x* is a dog	*Cxy*-*x* can command *y*
Ex-*x* is smoke	*Dxy*-*x* is done at (or by) *y*
Fx-*x* is fire	*Exy*-*x* shears *y*
Gx-*x* is glass	*Fxy*-*x* is fair for *y*
Hx-*x* is home	*Gxy*-*x* gathers *y*
Ix-*x* is ill	*Hxy*-*x* hears *y*
Jx-*x* is work	*Ixy*-*x* lives in *y*
Kx-*x* is a lining	*Jxy*-*x* is jack of *y*
Lx-*x* is a lamb	*Kxy*-*x* knows *y*
Mx-*x* is moss	*Lxy*-*x* likes *y*
Nx-*x* is good	*Mxy*-*x* is master of *y*
Ox-*x* is a fool	*Nxy*-*x* loses *y*
Px-*x* is a person	*Oxy*-*x* is judged by *y*
Qx-*x* is a place	*Pxyz*-*x* blows *y* to *z*
Rx-*x* rolls	*Qxy*-*x* keeps company with *y*
Sx-*x* is a stone	*Rxy*-*x* is like *y*
Tx-*x* is a trade	*Sxy*-*x* says *y*
Ux-*x* is a house	*Txy*-*x* should throw *y*
Vx-*x* is a woman	*Txyz*-*x* tempers *y* to *z*
Wx-*x* is wind	*Uxy*-*x* comes to *y*
Xx-*x* is a time	*Vxy*-*x* ventures *y*
Yx-*x* is a day	*Wxy*-*x* is at (or in) *y*
Zx-*x* waits	*Xxy*-*x* is parent of *y*

g-God

Formulas

1. $(x)[Dx \supset (\exists y)(Yy \cdot Byx)]$
2. $(x)[(\exists y)(Py \cdot Fxy) \supset (z)(Pz \supset Fxz)]$
3. $(x)[(Rx \cdot Sx) \supset (y)(My \supset \sim Gxy)]$
4. $(x)[(Px \cdot Axx) \supset (Agx)]$
5. $(x)[(Px \cdot Zx) \supset (y)(Uyx)]$

6. $(x)[Hx \supset (y)(Qy \supset \sim Ryx)]$
7. $(x)[(Px \cdot \sim Nxg) \supset (y)(\sim Nxy)]$
8. $(x)[(Px \cdot \sim Cxx) \supset (y)(\sim Cxy)]$
9. $(x)\{Cx \supset (\exists y)[(Ay \cdot Ky) \cdot Byx]\}$
10. $(x)[Px \supset (y)(Qxy \supset Oxy)]$
11. $(x)\{Qx \supset [(\exists y)(Ey \cdot Wyx) \supset (\exists z)(Fz \cdot Wzx)]\}$
12. $(x)\{[Px \cdot (y)(Ty \supset Jxy)] \supset (z)(Tz \supset \sim Mxz)\}$
13. $(x)\{[Px \cdot (\exists y)[(Gy \cdot Uy) \cdot Ixy]] \supset (z)(Sz \supset \sim Txz)\}$
14. $(x)\{[Px \cdot (y)(Lxy \supset Sxy)] \supset (\exists z)(Hxz \cdot \sim Lxz)\}$
15. $(x)\{[Wx \cdot (y)[Py \supset \sim(\exists z)(Nz \cdot Pxzy)]] \supset Ix\}$
16. $(x)\{[Px \cdot (y)(\sim Vxy)] \supset (z)(\sim Gxz)\}$
17. $(x)\{Vx \supset (y)[Xy \supset (\exists z)[(Jz \cdot Bzx) \cdot \sim Dzy]]\}$
18. $(x)\{[Lx \cdot (\exists y)(Py \cdot Eyx)] \supset (z)(Wz \supset Tgzx)\}$
19. $(x)\{Px \supset (\exists y)[Py \cdot (\exists z)(Bxzy)]\}$
20. $(x)\{Px \supset (\exists y)[Py \cdot (\exists z)(\sim Bxzy)]\}$
21. $(x)\{Px \supset (y)[Py \supset (z)(\sim Bxzy)]\}$
22. $(x)\{Px \supset (y)[Py \supset (\exists z)(\sim Bxzy)]\}$
23. $(x)\{(Nx \cdot Dx) \supset (y)[Py \supset (Myx \equiv Lxy)]\}$
24. $(x)[Px \supset (\exists y)(Py \cdot Xyx)] \cdot (\exists u)[Pu \cdot (v)(Pv \supset \sim Xuv)]$
25. $(x)\{[Qx \cdot (y)\{[(Py \cdot Wyx) \cdot (z)(\sim Kyz)] \supset By\}] \supset (u)\{[(Pu \cdot Wux) \cdot (v)(Kuv)] \supset Ou\}\}$

II. Symbolize the following sentences, in each case using the indicated symbols:

1. Dead men tell no tales. (*Dx*-*x* is dead, *Mx*-*x* is a man, *Tx*-*x* is a tale, *Txy*-*x* tells *y*.)
2. The early bird gets the worm. (*Ex*-*x* is early, *Bx*-*x* is a bird, *Wx*-*x* is a worm, *Gxy*-*x* gets *y*.)
3. A dead lion is more dangerous than a live dog. (*Lx*-*x* is a lion, *Ax*-*x* is alive, *Dx*-*x* is a dog, *Dxy*-*x* is more dangerous than *y*.)
4. Uneasy lies the head that wears a crown. (*Ux*-*x* lies uneasy, *Hx*-*x* is a head, *Cx*-*x* is a crown, *Wxy*-*x* wears *y*.)
5. Every rose has its thorn. (*Rx*-*x* is a rose, *Tx*-*x* is a thorn, *Hxy*-*x* has *y*.)
6. Anyone who consults a psychiatrist ought to have his head examined. (*Px*-*x* is a person, *Sx*-*x* is a psychiatrist, *Ox*-*x* ought to have his head examined, *Cxy*-*x* consults *y*.)
7. No one ever learns anything unless he teaches it to himself. (*Px*-*x* is a person, *Lxy*-*x* learns *y*, *Txyz*-*x* teaches *y* to *z*.)

8. Delilah wore a ring on every finger, and had a finger in every pie. (*d*-Delilah, *Rx*-*x* is a ring, *Fxy*-*x* is a finger of *y*, *Oxy*-*x* is on *y*, *Px*-*x* is a pie, *Ixy*-*x* is in *y*.)
9. The race is not always to the swift, nor the battle to the strong. (*Rx*-*x* is a race, *Sx*-*x* is swift, *Bx*-*x* is a battle, *Kx*-*x* is strong, *Wxy*-*x* wins *y*.)
10. Anyone who accomplishes anything will be envied by everyone. (*Px*-*x* is a person, *Axy*-*x* accomplishes *y*, *Exy*-*x* envies *y*.)
11. To catch a fish one must have some bait. (*Px*-*x* is a person, *Fx*-*x* is a fish, *Bx*-*x* is bait, *Cxy*-*x* catches *y*, *Hxy*-*x* has *y*.)
12. Every student does some problems, but no student does all of them. (*Sx*-*x* is a student, *Px*-*x* is a problem, *Dxy*-*x* does *y*.)
13. Any contestant who answers all the questions put to him will win any prize he chooses. (*Cx*-*x* is a contestant, *Qx*-*x* is a question, *Px*-*x* is a prize, *Axy*-*x* answers *y*, *Pxy*-*x* is put to *y*, *Wxy*-*x* wins *y*, *Cxy*-*x* chooses *y*.)
14. Every son has a father but not every father has a son. (*Px*-*x* is a person, *Mx*-*x* is male, *Pxy*-*x* is a parent of *y*.)
15. A person is maintaining a nuisance if he has a dog who barks at every stranger. (*Px*-*x* is a person, *Nx*-*x* is a nuisance, *Mxy*-*x* maintains *y*, *Dx*-*x* is a dog, *Bxy*-*x* barks at *y*, *Kxy*-*x* knows *y*, *Hxy*-*x* has *y*.)
16. A doctor has no scruples who treats a patient who has no ailment. (*Dx*-*x* is a doctor, *Sx*-*x* is a scruple, *Hxy*-*x* has *y*, *Px*-*x* is a patient, *Ax*-*x* is an ailment, *Txy*-*x* treats *y*.)
17. A doctor who treats a person who has every ailment has a job no one would envy him. (*Dx*-*x* is a doctor, *Px*-*x* is a person, *Txy*-*x* treats *y*, *Ax*-*x* is an ailment, *Hxy*-*x* has *y*, *Jx*-*x* is a job, *Exyz*-*x* envies *y* his *z*.)
18. If a farmer keeps only hens, none of them will lay eggs that are worth setting. (*Fx*-*x* is a farmer, *Kxy*-*x* keeps *y*, *Hx*-*x* is a hen, *Ex*-*x* is an egg, *Lxy*-*x* lays *y*, *Wx*-*x* is worth setting.)

In symbolizing the following, use only the abbreviations: *Px*-*x* is a person, *Sx*-*x* is a store, *Bxyz*-*x* buys *y* from *z*.

19. Everyone buys something from some store (or other).
20. There is a store from which everyone buys something (or other).
21. Some people make all their purchases from a single store.
22. No one buys everything that it sells from any store.
23. No one buys things from every store.

24. No store has everyone for a customer.
25. No store makes all its sales to a single customer.

II. ARGUMENTS INVOLVING RELATIONS

No new principles need be introduced to deal with relational arguments. The original list of valid argument forms, together with the strengthened method of Conditional Proof and our four quantification rules, enable us (if we have sufficient ingenuity) to construct a demonstration of the validity of every valid argument in which only individual variables are quantified and only truth-functional connectives occur.

However, a certain change of technique is advisable in working with arguments involving relations. In all our previous sample demonstrations, **UI** and **EI** were used to instantiate with respect to a variable different from any quantified in the premiss, and **UG** and **EG** were used to quantify with respect to a variable different from any which occurred free in the premiss. Our inferences were of the following forms:

$$\frac{(x)Fx}{\therefore Fy}, \quad \frac{(\exists x)Fx}{\therefore Fz}, \quad \frac{Fx}{\therefore (y)Fy}, \quad \frac{Fy}{\therefore (\exists w)Fw}.$$

But our statement of the quantification rules does not require that μ and ν be different variables; they may well be the same. And on the whole it is simpler (wherever it is legitimate) to instantiate with respect to the same variable that had been quantified, and to quantify with respect to the same variable that had been free in the premiss. Thus the above inferences may also take the following forms:

$$\frac{(x)Fx}{\therefore Fx}, \quad \frac{(\exists x)Fx}{\therefore Fx}, \quad \frac{Fx}{\therefore (x)Fx}, \quad \frac{Fy}{\therefore (\exists y)Fy}.$$

In this way instantiation is accomplished by simply dropping a quantifier, and generalization is accomplished by simply adding a quantifier. Of course our restrictions on the quantification rules must still be observed. For example. where we have two premisses '$(\exists x)Fx$' and '$(\exists x) \sim Fx$', we can instantiate with respect to one by simply dropping the quantifier, but when this is done, if **EI** is subsequently used on the other, a new variable must be used instead of 'x', for the latter will already have a free occurrence in the proof under construction. Of course we remain perfectly free to use **UI** to instantiate with respect to any particular variable or constant we choose. The preceding remarks can be illustrated by constructing a demonstration of validity for the argument

There is a man whom all men despise.
Therefore at least one man despises himself.

Its symbolic translation and proof, using 'Mx' and 'Dxy' to abbreviate 'x is a man' and 'x despises y' may be written as follows:

1. $(\exists x)[Mx \cdot (y)(My \supset Dyx)] / \therefore (\exists x)(Mx \cdot Dxx)$	
2. $Mx \cdot (y)(My \supset Dyx)$	1, **EI**
3. $(y)(My \supset Dyx)$	2, Simp.
4. $Mx \supset Dxx$	3, **UI**
5. Mx	2, Simp.
6. Dxx	4, 5, M.P.
7. $Mx \cdot Dxx$	5, 6, Conj.
8. $(\exists x)(Mx \cdot Dxx)$	7, **EG**

In the foregoing proof, the only use of a quantification rule which was accompanied by a change of variable was the use of **UI** in going from step 3 to step 4, which was done because we needed the expression 'Dxx' thus obtained.

Another sample demonstration will be given, this time to establish the validity of the third specimen argument stated at the beginning of the present chapter. Its premiss, 'All horses are animals' will be symbolized as '$(x)(Ex \supset Ax)$', where 'Ex' and 'Ax' abbreviate 'x is a horse' and 'x is an animal', respectively. In its conclusion

The head of a horse is the head of an animal

the word 'the' has the same sense that it does in such propositions as 'The whale is a mammal' or 'The burnt child dreads the fire'.

We may paraphrase it therefore first as

All heads of horses are heads of animals.

then as

$(x)[(x \text{ is the head of a horse}) \supset (x \text{ is the head of an animal})]$.

and finally, writing 'Hxy' for 'x is the head of y', we may express the conclusion by the formula

$$(x)[(\exists y)(Ey \cdot Hxy) \supset (\exists y)(Ay \cdot Hxy)].$$

Once it is symbolized, the argument is easily proved valid by the techniques already available:

1. $(x)(Ex \supset Ax) / \therefore (x)[(\exists y)(Ey \cdot Hxy) \supset (\exists y)(Ay \cdot Hxy)]$	
→2. $(y) \sim(Ay \cdot Hxy)$	
3. $\sim(Ay \cdot Hxy)$	2, **UI**
4. $\sim Ay \vee \sim Hxy$	3, De M.
5. $Ay \supset \sim Hxy$	4, Impl.
6. $Ey \supset Ay$	1, **UI**
7. $Ey \supset \sim Hxy$	6, 5, H.S.
8. $\sim Ey \vee \sim Hxy$	7, Impl.
9. $\sim(Ey \cdot Hxy)$	8, De M.
10. $(y) \sim(Ey \cdot Hxy)$	9, **UG**
11. $(y) \sim(Ay \cdot Hxy) \supset (y) \sim(Ey \cdot Hxy)$	2–10, C.P.
12. $\sim(y) \sim(Ey \cdot Hxy) \supset \sim(y) \sim(Ay \cdot Hxy)$	11, Trans.
13. $(\exists y)(Ey \cdot Hxy) \supset (\exists y)(Ay \cdot Hxy)$	12, **QN**
14. $(x)[(\exists y)(Ey \cdot Hxy) \supset (\exists y)(Ay \cdot Hxy)]$	13, **UG**

Again, the only time a change of variables accompanied the use of a quantification rule (in step 6) was when the change of variable was needed for subsequent inferences.

The first specimen argument presented in this chapter, which dealt with the relation of *being older than*, raises a new problem, which will be discussed in the following section.

EXERCISES

Construct a formal proof of validity for each of the following arguments:

1. Whoever supports Ickes will vote for Jones. Anderson will vote for no one but a friend of Harris. No friend of Kelly has Jones for a friend. Therefore, if Harris is a friend of Kelly, Anderson will not support Ickes. (*Sxy-x* supports *y*, *Vxy-x* votes for *y*, *Fxy-x* is a friend of *y*, *a*-Anderson, *i*-Ickes, *j*-Jones, *h*-Harris, *k*-Kelly.)
2. Whoever belongs to the Country Club is wealthier than any member of the Elks Lodge. Not all who belong to the Country Club are wealthier than all who do not belong. Therefore not everyone belongs either to the Country Club or the Elks Lodge. (*Cx-x* belongs to the Country Club, *Ex-x* belongs to the Elks Lodge, *Px-x* is a person, *Wxy-x* is wealthier than *y*.)
3. All circles are figures. Therefore all who draw circles draw figures. (*Cx-x* is a circle, *Fx-x* is a figure, *Dxy-x* draws *y*.)
4. There is a professor who is liked by every student who likes any professor at all. Every student likes some professor or other. Therefore there is a professor who is liked by all students. (*Px-x* is a professor, *Sx-x* is a student, *Lxy-x* likes *y*.)
5. Only a fool would lie about one of Bill's fraternity brothers to him. A classmate of Bill's lied about Al to him. Therefore if none of Bill's classmates are fools, then Al is not a fraternity brother of Bill. (*Fx-x* is a fool, *Lxyz-x* lies about *y* to *z*, *Cxy-x* is a classmate of *y*, *Bxy-x* is a fraternity brother of *y*, *a*-Al, *b*-Bill.)
6. It is a crime to sell an unregistered gun to anyone. All the weapons that Red owns were purchased by him from either Lefty or Moe. So if one of Red's weapons is an unregistered gun, then if Red never bought anything from Moe, Lefty is a criminal. (*Rx-x* is registered, *Gx-x* is a gun, *Cx-x* is a criminal, *Wx-x* is a weapon, *Oxy-x* owns *y*, *Sxyz-x* sells *y* to *z*, *r*-Red, *l*-Lefty, *m*-Moe.)
7. No one respects a person who does not respect himself. No one will hire a person he does not respect. Therefore a person who respects no one will never be hired by anybody. (*Px-x* is a person, *Rxy-x* respects *y*, *Hxy-x* hires *y*.)
8. Everything on my desk is a masterpiece. Anyone who writes a masterpiece is a genius. Someone very obscure wrote some of the novels on my desk. Therefore some very obscure person is a genius. (*Dx-x* is on my desk, *Mx-x* is a masterpiece, *Px-x* is a person, *Gx-x* is a genius, *Ox-x* is very obscure, *Nx-x* is a novel, *Wxy-x* wrote *y*.)
9. Any book which is approved by all critics is read by every literary person. Anyone who reads anything will talk about it. A critic will approve any book written by any person who flatters him. Therefore if someone flatters every critic then any book he writes will be talked about by all literary persons. (*Bx-x* is a book, *Cx-x* is a critic, *Lx-x* is literary, *Px-x* is a person, *Axy-x* approves *y*, *Rxy-x* reads *y*, *Txy-x* talks about *y*, *Fxy-x* flatters *y*, *Wxy-x* writes *y*.)
10. A work of art which tells a story can be understood by everyone. Some religious works of art have been created by great artists. Every religious work of art tells an inspirational story. Therefore if some people admire only what they cannot understand, then some artists' creations will not be admired by everyone. (*Ax-x* is an artist, *Gx-x* is great, *Px-x* is a person, *Sx-x* is a story, *Ix-x* is inspirational, *Rx-x* is religious, *Wx-x* is a work of art, *Cxy-x* creates *y*, *Axy-x* admires *y*, *Txy-x* tells *y*, *Uxy-x* can understand *y*.)

III. SOME PROPERTIES OF RELATIONS

There are a number of interesting properties that relations themselves may possess. We shall consider only a few of the more familiar ones, and our discussion will be confined to properties of *dyadic* relations.

Dyadic relations may be characterized as *symmetrical*, *asymmetrical*, or *non-symmetrical*. Various symmetrical relations are designated by the phrases: 'is next to', 'is married to', and 'has the same weight as'. A *symmetrical* relation is one such that if one individual has that relation to a second individual, then the second individual must have that relation to the first. A propositional function '*Rxy*' designates a symmetrical relation if and only if

$$(x)(y)(Rxy \supset Ryx).$$

On the other hand, an *asymmetrical* relation is one such that if one individual has that relation to a second individual, then the second individual *cannot* have that relation to the first. Various asymmetrical relations are designated by the phrases: 'is north of', 'is parent of', and 'weighs more than'. A propositional function '*Rxy*' designates an asymmetrical relation if and only if

$$(x)(y)(Rxy \supset \sim Ryx).$$

Not all relations are either symmetrical or asymmetrical, however. If one individual loves a second, or is a brother of a second, or weighs no more than a second, it does not follow that the second loves the first, or is a brother to the first (possibly being a sister instead), or weighs no more than the first. Nor does it follow that the second does *not* love the first, or is *not* a brother to him, or *does* weigh more than the first. Such relations as these are *non-symmetrical*, and are defined as those which are neither symmetrical nor asymmetrical.

Dyadic relations may also be characterized as *transitive*, *intransitive*, or *non-transitive*. Various transitive relations are designated by the phrases: 'is north of', 'is an ancestor of', and 'weighs the same as'. A *transitive* relation is one such that if one individual has it to a second, and the second to a third, then the first must have it to the third. A propositional function 'Rxy' designates a transitive relation if and only if

$$(x)(y)(z)[(Rxy \cdot Ryz) \supset Rxz].$$

An *intransitive* relation, on the other hand, is one such that if one individual has it to a second, and the second to a third, then the first *cannot* have it to the third. Some intransitive relations are designated by the phrases: 'is mother of', 'is father of', and 'weighs exactly twice as much as'. A propositional function 'Rxy' designates an intransitive relation if and only if

$$(x)(y)(z)[(Rxy \cdot Ryz) \supset \sim Rxz].$$

Not all relations are either transitive or intransitive. We define a *non-transitive* relation as one which is neither transitive nor intransitive; examples of non-transitive relations are designated by: 'loves', 'is discriminably different from', and 'has a different weight than'.

Finally, relations may be *reflexive*, *irreflexive*, or *non-reflexive*. Various definitions of these properties have been proposed by different authors, and there seems to be no standard terminology established. It is convenient to distinguish between reflexivity and total reflexivity. A relation is *totally reflexive* if every individual has that relation to itself. For example, the phrase 'is identical with' designates the totally reflexive relation of identity. A propositional function 'Rxy' designates a totally reflexive relation if and only if

$$(x)Rxx.$$

On the other hand, a relation is said to be *reflexive* if any individuals which stand in that relation to each other also have that relation to themselves. Obvious examples of reflexive relations are designated by the phrases: 'has the same color hair as', 'is the same age as', and 'is a contemporary of'. A propositional function 'Rxy' designates a reflexive relation if and only if

$$(x)(y)[(Rxy \supset (Rxx \cdot Ryy)].$$

It is obvious that all totally reflexive relations are reflexive.

An irreflexive relation is one which no individual has to itself. A propositional function 'Rxy' designates an irreflexive relation if and only if

$$(x) \sim Rxx.$$

Examples of irreflexive relations are common indeed; the phrases: 'is north of', 'is married to', and 'is parent of' all designate irreflexive relations. Relations which are neither reflexive nor irreflexive are said to be *non-reflexive*. The phrases: 'loves', 'hates', and 'criticizes' designate non-reflexive relations.

Relations may have various combinations of the properties described. The relation of *weighing more than* is asymmetrical, transitive, and irreflexive, while the relation of *having the same weight as* is symmetrical, transitive, and reflexive. However, some properties entail the presence of others. For example, all asymmetrical relations must be irreflexive, as can easily be demonstrated. Let 'Rxy' designate any asymmetrical relation; then by definition:

1. $(x)(y)(Rxy \supset \sim Ryx)$.

From this premiss we can deduce that R is irreflexive, that is, that $(x)\sim Rxx$:

2. $(y)(Rxy \supset \sim Ryx)$	1, **UI**
3. $Rxx \supset \sim Rxx$	2, **UI**
4. $\sim Rxx \vee \sim Rxx$	3, Impl.
5. $\sim Rxx$	4, Taut.
6. $(x) \sim Rxx$	5, **UG**

Other logical connections among these properties of relations are easily stated and proved, but our interest lies in another direction.

The relevance of these properties to relational arguments is easily seen. An argument to which one of them is relevant might be stated thus:

> Tom has the same weight as Dick.
> Dick has the same weight as Harry.
> The relation of *having the same weight as* is transitive.
> ―――――――――――――――――――――
> Therefore Tom has the same weight as Harry.

When it is translated into our symbolism as

$$\begin{array}{l} Wtd \\ Wdh \\ \underline{(x)(y)(z)[(Wxy \cdot Wyz) \supset Wxz]} \\ \therefore Wth \end{array}$$

the method of its validation is immediately obvious. We said that the argument 'might' be stated in the way indicated. But such a statement of the argument would be the rare exception rather than the rule. The ordinary way of propounding such an argument would be to state only the first two premisses and the conclusion, on the grounds that *everyone knows* that *having the same weight as* is a transitive relation. Relational arguments are often used, and many of them depend essentially on the transitivity, or symmetry, or one of the other properties of the relations involved. But *that* the relation in question *has* the relevant property is seldom—if ever—stated explicitly as a premiss. The reason is easy to see. In most discussions a large body of propositions can be presumed to be common knowledge. The majority of speakers and writers save themselves trouble

by not repeating well-known and perhaps trivially true propositions which their hearers or readers can perfectly well be expected to supply for themselves. An argument which is incompletely expressed, part of it being 'understood', is an *enthymeme.*

Because it is incomplete, an enthymeme must have its suppressed premiss or premisses taken into account when the problem arises of testing its validity. Where a necessary premiss is missing, the inference is technically invalid. But where the unexpressed premiss is easily supplied and obviously true, in all fairness it ought to be included as part of the argument in any evaluation of it. In such a case one assumes that the maker of the argument did have more 'in mind' than he stated explicitly. In most cases there is no difficulty in supplying the tacit premiss that the speaker intended but did not express. Thus the first specimen argument stated at the beginning of this chapter:

> Al is older than Bill.
> Bill is older than Charlie.
> ―――――――――――――――――――――
> Therefore Al is older than Charlie.

ought to be counted as valid, since it becomes so when the trivially true proposition that *being older than* is a transitive relation, is added as an auxiliary premiss. When the indicated missing premiss is supplied, a formal proof of the argument's validity is very easily set down.

Of course premisses other than relational ones are often left unexpressed. For example, in the argument

> Any horse can outrun any dog. Some greyhounds can outrun any rabbit. Therefore any horse can outrun any rabbit.

not only is the needed premiss about the transitivity of *being able to outrun* left unexpressed, but also the non-relational premiss that all greyhounds are dogs. When these are added—and they are certainly not debatable issues—the validity of the argument can be demonstrated as follows:

1.	$(x)[Hx \supset (y)(Dy \supset Oxy)]$	premisses/ ∴ $(x)[Hx \supset (z)(Rz \supset Oxz)]$
2.	$(\exists y)[Gy \cdot (z)(Rz \supset Oyz)]$	
3.	$(x)(y)(z)[(Oxy \cdot Oyz) \supset Oxz]$	additional premisses
4.	$(y)(Gy \supset Dy)$	
→5.	Hx	
6.	$Hx \supset (y)(Dy \supset Oxy)$	1, **UI**
7.	$(y)(Dy \supset Oxy)$	6, 5, M.P.
8.	$Gy \cdot (z)(Rz \supset Oyz)$	2, **EI**
9.	Gy	8, Simp.
10.	$Gy \supset Dy$	4, **UI**
11.	Dy	10, 9, M.P.
12.	$Dy \supset Oxy$	7, **UI**
13.	Oxy	12, 11, M.P.
14.	$(z)(Rz \supset Oyz)$	8, Simp.
→15.	Rz	
16.	$Rz \supset Oyz$	14, **UI**
17.	Oyz	16, 15, M.P.
18.	$Oxy \cdot Oyz$	13, 17, Conj.
19.	$(y)(z)[(Oxy \cdot Oyz) \supset Oxz]$	3, **UI**
20.	$(z)[(Oxy \cdot Oyz) \supset Oxz]$	19, **UI**
21.	$(Oxy \cdot Oyz) \supset Oxz$	20, **UI**
22.	Oxz	21, 18, M.P.
23.	$Rz \supset Oxz$	15–22, C.P.
24.	$(z)(Rz \supset Oxz)$	23, **UG**
25.	$Hx \supset (z)(Rz \supset Oxz)$	5–24, C.P.
26.	$(x)[Hx \supset (z)(Rz \supset Oxz)]$	25, **UG**

Missing premisses are not always so easily noticed and supplied as in the present example. When it is not so obvious which necessary premisses are missing from an enthymematically expressed argument, then in beginning a proof of its validity it is a good policy to leave a little space just below the given premisses, in which additional premisses can be written when need arises for their use. The only point to be stressed is that no statement which is as doubtful or debatable as the argument's own conclusion is to be admitted as a supplementary premiss, for in a valid argument which is enthymematically stated only the sheerest platitudes should be left unexpressed for the hearer or reader to fill in for himself.

EXERCISES

Prove the validity of the following enthymemes—adding only obviously true premisses where necessary:

1. A Cadillac is more expensive than any low-priced car. Therefore no Cadillac is a low-priced car. (*Cx*-*x* is a Cadillac, *Lx*-*x* is a low-priced car, *Mxy*-*x* is more expensive than *y*.)
2. Alice is Betty's mother. Betty is Charlene's mother. Therefore if Charlene loves only her mother then she does not love Alice. (*a*-Alice, *b*-Betty, *c*-Charlene, *Mxy*-*x* is mother of *y*, *Lxy*-*x* loves *y*.)
3. Any man on the first team can outrun every man on the second team. Therefore no man on the second team can outrun any man on the first team. (*Fx*-*x* is a man on the first team, *Sx*-*x* is a man on the second team, *Oxy*-*x* can outrun *y*.)
4. Every boy at the party danced with every girl who was there. Therefore every girl at the party danced with every boy who was there. (*Bx*-*x* is a boy, *Gx*-*x* is a girl, *Px*-*x* was at the party, *Dxy*-*x* danced with *y*.)
5. Anyone is unfortunate who bears the same name as a person who commits a crime. Therefore anyone who commits a burglary is unfortunate. (*Px*-*x* is a person, *Ux*-*x* is unfortunate, *Cx*-*x* is a crime, *Bx*-*x* is a burglary, *Cxy*-*x* commits *y*, *Nxy*-*x* bears the same name as *y*.)
6. All the watches sold by Kubitz are made in Switzerland. Anything made in a foreign country has a tariff paid on it. Anything on which a tariff was paid costs its purchaser extra. Therefore it will cost anyone extra who buys a watch from Kubitz. (*Wx*-*x* is a watch, *Tx*-*x* has a tariff paid on it, *Fx*-*x* is a foreign country, *Cxy*-*x* costs *y* extra, *Mxy*-*x* is made in *y*, *Bxyz*-*x* buys *y* from *z*, *s*-Switzerland, *k*-Kubitz.)
7. Vacant lots provide no income to their owners. Anyone who owns real estate must pay taxes on it. Therefore anyone who owns a vacant lot must pay taxes on something which provides no income to him. (*Vx*-*x* is a vacant lot, *Rx*-*x* is real estate, *Ixy*-*x* provides income to *y*, *Txy*-*x* pays taxes on *y*, *Oxy*-*x* owns *y*.)
8. All admirals wear uniforms having gold buttons. Therefore some naval officers wear clothes which have metal buttons. (*Ax*-*x* is an admiral, *Ux*-*x* is a uniform, *Gx*-*x* is gold, *Bx*-*x* is a button, *Nx*-*x* is a naval officer, *Cx*-*x* is clothing, *Mx*-*x* is metal, *Wxy*-*x* wears *y*, *Hxy*-*x* has *y*.)