

... für alle  $a \in R$  gilt  $a \cdot e = a$  und  $e \cdot a = a$

$$e(1+a) = 0$$

$$\uparrow$$

$$e=0$$

$$a + e + a \cdot e = a$$

$$e \cdot a = a \cdot e = a$$

... für alle  $a \in R$  gilt  $a \cdot e = a$  und  $e \cdot a = a$

②  $\Rightarrow$   $a \cdot b = a$  für alle  $a \in R$

$$a \cdot b = a \cdot a = a$$

... für alle  $a \in R$  gilt  $a \cdot b = a$  und  $a \cdot a = a$

③  $\Rightarrow$   $a \cdot b = a$  für alle  $a \in R$

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①

$$u_{12} = \sum_{i=1}^n x_i^2 = 21$$

$$(a, b) = 2 = (2, 1)$$

על ידי שימוש במשוואות אלו נקבל:

② נניח שיש פתרון כלשהו למערכת:

$$\begin{cases} 10k \equiv 0 \pmod{12} \\ 9k \equiv 0 \pmod{12} \\ 8k \equiv 0 \pmod{12} \end{cases} \implies \begin{cases} g = 10 \\ h = 9 \\ z = 8 \end{cases}$$

$$x \equiv 0 \pmod{12} \implies x = 12j$$

נציב את  $x = 12j$  במשוואות:

$$12j \equiv 0 \pmod{12} \implies 0 \equiv 0 \pmod{12}$$

כלומר המשוואה הראשונה היא תמיד נכונה.

$$12j \equiv 0 \pmod{9} \implies 3j \equiv 0 \pmod{3} \implies j \equiv 0 \pmod{3}$$

$$12j \equiv 0 \pmod{8} \implies 3j \equiv 0 \pmod{2} \implies j \equiv 0 \pmod{2}$$

לכן  $j$  חייב להיות זוגי ונתון ל-3.

$$\begin{aligned} z &= 10 \\ h &= 9 \\ k &= 8 \\ g &= 7 \end{aligned}$$

$$z = 10 \implies 10j \equiv 0 \pmod{12}$$

$$\text{②} \quad \frac{10j}{12} = 2$$

$$10j - 12k = 0$$



$$5j - 6k = 0$$

$$5j = 6k \implies j = \frac{6k}{5}$$

$$j = \frac{6k}{5} \implies j = 6m \implies k = 5m$$

$$x = 12j = 72m$$

②

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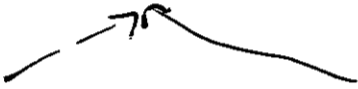
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③

$$a^2 + b^2 = a^2 + b^2$$

$$a^2 + b^2 = a^2 + b^2$$



$$e = (ab)^2 = a^2 b^2$$

②

$$a^2 = e$$

$$b^2 = e$$

②

(c) ...

$$3 + 3 + 3 = 24$$

②

...

...

$$a^2 = a * a = a^2$$

...

③

...

...

...

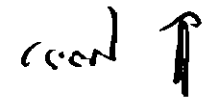
③

⑨

$$b^2 = a^2$$



$$a^3 b = a^2 b^2$$



$$b^3 = a^3$$



$$b^4 = a^4$$



$$a^5 b^2 = a^4 b^3$$

$$(a^5 b^2)^2 = a^{10} b^4$$

$$(a^5 b^2)^3 = a^{15} b^6$$

$$(a^5 b^2)^4 = a^{20} b^8$$

$$b^3 = a^3$$

$$a^6 b^3 = a^3 b^6$$

$$(a^6 b^3)^2 = a^{12} b^6$$

$$(a^6 b^3)^3 = a^{18} b^9$$

$$(a^6)^4 = a^{24}$$

may not be

may not be