פתרון

$$f(x)=\frac{1+x}{(1-x^2)^2}=\frac{1+x}{(1+x)^2(1-x)^2}=\frac{1}{(1+x)(1-x)^2}=$$

$$=\frac{A}{1+x}+\frac{B}{(1-x)}+\frac{C}{(1-x)^2}=\frac{A(1-x)^2+B(1-x^2)+C(1+x)}{(1+x)(1-x)^2}$$

$x=1$: $\quad 2C=1 \Rightarrow \boxed{C=\frac{1}{2}}$

$x=-1$: $\quad 4A=1 \Rightarrow \boxed{A=\frac{1}{4}}$

$x=0$: $\quad A+B+C=1 \Rightarrow \boxed{B=\frac{1}{4}}$

$$\Rightarrow f(x)=\frac{1}{4}\left[\frac{1}{1+x}+\frac{1}{1-x}+\frac{2}{(1-x)^2}\right]$$

טור גאומטרי: $\quad \frac{1}{1-x}=\sum_0^\infty x^n \quad (|x|<1)$

$$\Rightarrow \frac{1}{1+x}=\sum_0^\infty(-x)^n=\sum_0^\infty(-1)^n x^n$$

$$\frac{1}{(1-x)^2}=\left(\frac{1}{1-x}\right)'=\left(\sum_0^\infty x^n\right)'=\sum_1^\infty n x^{n-1}\underset{n\to n+1}{=}\sum_0^\infty(n+1)x^n$$

כלל הרכנסור קיים.

כי $0=(x^0)'$.

$$\Rightarrow f(x)=\frac{1}{4}\left[\sum_0^\infty x^n+\sum_0^\infty(-1)^n x^n+2\sum_{n=0}^\infty(n+1)x^n\right]$$

$$f(x)=\sum_{n=0}^\infty\frac{1+(-1)^n+2(n+1)}{4}x^n=\boxed{\sum_{n=0}^\infty\frac{2n+3+(-1)^n}{4}x^n}$$