

גבול ונגזרת של פונקציה למכינה.

פתרונות ותשובות.

$$1. \lim_{y \rightarrow 2} \frac{y^2 - 5y + 6}{y - 2} = \left\{ \frac{0}{0} \right\} = \lim_{y \rightarrow 2} \frac{(y-2)(y-3)}{y-2} = \lim_{y \rightarrow 2} (y-3) = -1 \quad (\aleph)$$

$$\lim_{x \rightarrow 4} \frac{x-4}{x^2 - 5x + 4} = \lim_{x \rightarrow 4} \frac{x-4}{(x-4)(x-1)} = \frac{1}{3} \quad (\beth)$$

$$\lim_{x \rightarrow 2} \frac{2x-4}{x^3 - 2x^2} = \lim_{x \rightarrow 2} \frac{2(x-2)}{(x-2)x^2} = \frac{1}{2} \quad (\gamma)$$

$$\lim_{x \rightarrow 0} \frac{(1+x)^4 - 1}{x} = \left\{ \frac{0}{0} \right\} = \lim_{x \rightarrow 0} \frac{1 + 4x + 6x^2 + 4x^3 + x^4 - 1}{x} = 4 \quad (\delta)$$

$$\lim_{x \rightarrow 1} \frac{5x^2 - x - 4}{6x^3 - x - 5} = \lim_{x \rightarrow 1} \frac{(5x+4)(x-1)}{(6x^2 + 6x + 5)(x-1)} = \frac{9}{17} \quad (\eta)$$

$$2. \lim_{x \rightarrow 0} \frac{\sqrt{1+x} - 1}{x} = \lim_{x \rightarrow 0} \frac{(\sqrt{1+x} - 1)(\sqrt{1+x} + 1)}{x(\sqrt{1+x} + 1)} = \lim_{x \rightarrow 0} \frac{x}{x(\sqrt{1+x} + 1)} = \frac{1}{2} \quad (\aleph)$$

$$\lim_{x \rightarrow 7} \frac{2 - \sqrt{x-3}}{x^2 - 49} = \left\{ \frac{0}{0} \right\} = \lim_{x \rightarrow 7} \frac{(2 - \sqrt{x-3})(2 + \sqrt{x-3})}{x^2 - 49} = \quad (\beth)$$

$$= \lim_{x \rightarrow 7} \frac{7-x}{(x^2 - 49)(2 + \sqrt{x-3})} = -\lim_{x \rightarrow 7} \frac{1}{(x+7)(2 + \sqrt{x-3})} = -\frac{1}{56}$$

$$\lim_{x \rightarrow 9} \frac{9-x}{3-\sqrt{x}} = \lim_{x \rightarrow 9} \frac{(3-\sqrt{x})(3+\sqrt{x})}{3-\sqrt{x}} = \lim_{x \rightarrow 9} (3+\sqrt{x}) = 6 \quad (\gamma)$$

$$\lim_{x \rightarrow 0} \frac{\sqrt{2-x} - \sqrt{2}}{x} = \lim_{x \rightarrow 0} \frac{(\sqrt{2-x} - \sqrt{2})(\sqrt{2-x} + \sqrt{2})}{x(\sqrt{2-x} + \sqrt{2})} = \lim_{x \rightarrow 0} \frac{-x}{x(\sqrt{2-x} + \sqrt{2})} = -\frac{1}{2\sqrt{2}} \quad (\delta)$$

$$\lim_{x \rightarrow 2^+} \frac{x-2}{2-\sqrt{2x}} = -\frac{1}{\sqrt{2}} \lim_{x \rightarrow 2^+} \frac{2-x}{\sqrt{2}-\sqrt{x}} = -\frac{1}{\sqrt{2}} \lim_{x \rightarrow 2^+} (\sqrt{2} + \sqrt{x}) = -2 \quad (\eta)$$

$$3. \lim_{h \rightarrow 0} \frac{2(3+h)^2 - (3+h) + 5 - 20}{h} = \lim_{h \rightarrow 0} \frac{11h + 2h^2}{h} = 11 \quad (\aleph)$$

$$\lim_{h \rightarrow 0} \frac{3+h - \frac{1}{3+h} - \frac{8}{3}}{h} = \lim_{h \rightarrow 0} \frac{3(3+h)^2 - 3 - 8(3+h)}{3h(3+h)} = \lim_{h \rightarrow 0} \frac{10h + 3h^2}{3h(3+h)} = \frac{10}{9} \quad (\beth)$$

$$4. \lim_{h \rightarrow 0} \frac{1}{h} (\sqrt{4+h} - \sqrt{4}) = \lim_{h \rightarrow 0} \frac{1}{h} \left(\frac{(\sqrt{4+h} - \sqrt{4})(\sqrt{4+h} + \sqrt{4})}{\sqrt{4+h} + \sqrt{4}} \right) = \lim_{h \rightarrow 0} \frac{1}{h} \left(\frac{h}{\sqrt{4+h} + 2} \right) = \frac{1}{4}$$

פונקציה $f(x) = \sqrt{x}$, $x_0 = 4$

$$\lim_{h \rightarrow 0} \frac{18-18}{h} = 0, D(f) = (-\infty, +\infty), D(f') = (-\infty, +\infty) \quad (\text{א}) \quad .5$$

$$\lim_{h \rightarrow 0} \frac{\sqrt{6-(x+h)} - \sqrt{6-x}}{h} = \lim_{h \rightarrow 0} \frac{-h}{h(\sqrt{6-(x+h)} + \sqrt{6-x})} = -\frac{1}{2\sqrt{6-x}}, \quad (\text{ב})$$

$$D(f) = (-\infty, 6], D(f') = (-\infty, 6)$$

$$\lim_{h \rightarrow 0} \frac{x+h - \frac{2}{x+h} - x + \frac{2}{x}}{h} = \lim_{h \rightarrow 0} \frac{hx(x+h) + 2h}{hx(x+h)} = 1 + \frac{2}{x^2}, D(f) = (-\infty, 0) \cup (0, +\infty), \quad (\text{ג})$$

$$D(f') = (-\infty, 0) \cup (0, +\infty)$$

$$\lim_{h \rightarrow 0} \frac{(x+h)^3 - x^3}{h} = \lim_{h \rightarrow 0} \frac{3x^2h + 3xh^2 + h^3}{h} = 3x^2, D(f) = \mathbf{R}, D(f') = \mathbf{R} \quad (\text{ד})$$

$$\frac{2^x(x \ln x \cdot \ln 2 + 1)}{x \ln 2} \quad (\text{א}) \quad , -\frac{x^2 \sin x + 2 \sin x + 2x \cos x}{(x^2 + 2)^2} \quad (\text{ב}) \quad , 6x - \frac{5}{2\sqrt{x}} \quad (\text{ג}) \quad .6$$

$$\frac{1}{1 - \sin x} \quad (\text{ד})$$

$$-\sin(\sin^2 x) \cdot \sin 2x \quad (\text{א}) \quad , \frac{1}{\sqrt{(1+x)(1-x)^3}} \quad (\text{ב}) \quad , \frac{x(3x^2 - 2a^2)}{\sqrt{x^2 - a^2}} \quad (\text{ג}) \quad .7$$

$$\frac{1}{x \ln x} \quad (\text{ד}) \quad , 3x^2 \ln x^2 + 2x^2 \quad (\text{ה}) \quad , \frac{-9(4x+5)}{2(2x^2 + 5x)^{\frac{5}{2}}} \quad (\text{ו})$$

$$, y = -\frac{x}{11} + \frac{267}{11} \text{ הנורמל } , y = f(3) + f'(3)(x-3), y = 11x - 9 \text{ המשיק } \quad (\text{א}) \quad .8$$

$$, y = -\frac{x}{12} - \frac{49}{6} \text{ הנורמל } , y = f(-2) + f'(-2)(x+2), y = 12x + 16 \text{ המשיק } \quad (\text{ב})$$

$$, y = -x \text{ הנורמל } , y = f(0) + f'(0)x, y = x \text{ המשיק } \quad (\text{ג})$$

$$. y = -x + 1 \text{ הנורמל } , y = f(1) + f'(1)(x-1), y = x - 1 \text{ המשיק } \quad (\text{ד})$$

בהצלחה בתואר!