בס״ד

אינטגרל לא מסוים:

שיטת ההצבה:

- $$\int (2x+3)^{10}\,dx = \frac{1}{2}\int t^{10}\,dt = \frac{1}{2}\cdot\frac{t^{11}}{11}+c = \frac{(2x+3)^{11}}{22}+c$$

$t=2x+3,\ \frac{dt}{2}=dx$

- $$\int \frac{1}{(-4x+7)^8}\,dx = \int (-4x+7)^{-8}\,dx = -\frac{1}{4}\int t^{-8}\,dt = -\frac{1}{4}\cdot\frac{t^{-7}}{-7}+c = \frac{(-4x+7)^{-7}}{28}+c$$

$t=-4x+7,\ -\frac{1}{4}dt=dx$

- $$\int e^{3x}\,dx = \frac{1}{3}\int e^t\,dt = \frac{e^t}{3}+c = \frac{e^{3x}}{3}+c$$

$t=3x,\ \frac{1}{3}dt=dx$

- $$\int xe^{x^2}\,dx = \int e^t\cdot\frac{1}{2}dt = \frac{1}{2}e^t+c = \frac{1}{2}e^{x^2}+c$$

$t=x^2,\ dt=2x\,dx$

- $$\int \tan(x)\,dx = \int\frac{\sin x}{\cos x}\,dx = -\int\frac{1}{t}\,dt = -\ln|t|+c = -\ln|\cos x|+c$$

$t=\cos x,\ dt=-\sin(x)\,dx$

- $$\int \sin(x^2)\cdot 2x\,dx = \int \sin(t)\,dt = -\cos(t)+c = -\cos(x^2)+c$$

$t=x^2,\ dt=2x\,dx$

בס"ד

• $\int \sin^{42}(x)\cdot\cos(x)\,dx \overset{\uparrow}{=} \int t^{42}\,dt = \frac{t^{43}}{43}+C = \frac{(\sin(x))^{43}}{43}+C$

$t=\sin x,\ dt=\cos x\,dx$

• $\int \frac{1}{x}\cos(\ln|x|)\,dx \overset{\uparrow}{=} \int \cos(t)\,dt = \sin t+C = \sin(\ln|x|)+C$

$t=\ln|x|,\ dt=\frac{1}{x}dx$

•• $\int x^3(3x^2-1)^{17}\,dx \overset{\uparrow}{=} \int x^3 t^{17}\frac{dt}{6x} = \frac{1}{6}\int x^2 t^{17}\,dt =$

$t=3x^2-1,\ dt=6x\,dx,\ \frac{dt}{6x}=dx$

$x^2=\frac{t+1}{3}$

$= \frac{1}{6}\cdot\int \frac{t+1}{3}\cdot t^{17}\,dt = \frac{1}{18}\int t^{18}+t^{17}\,dt = \frac{1}{18}\left[\frac{t^{19}}{19}+\frac{t^{18}}{18}+C\right] =$

$= \frac{1}{18}\left[\frac{(3x^2-1)^{19}}{19}+\frac{(3x^2-1)^{18}}{18}+C\right]$

•• $\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \int \frac{1}{\sqrt{a^2}\cdot\sqrt{1-\frac{x^2}{a^2}}} = \frac{1}{a}\int \frac{1}{\sqrt{1-(\frac{x}{a})^2}} =$

$\overset{\uparrow}{=} \frac{1}{a}\int \frac{1}{\sqrt{1-t^2}}\cdot a\,dt = \int \frac{1}{\sqrt{1-t^2}}\,dt = \arcsin(t)+C =$

$t=\frac{x}{a},\ dt=\frac{1}{a}dx,\ a\cdot dt=dx$

$= \arcsin(\frac{x}{a})+C$

•• $\int \frac{e^x}{e^{2x}+1}\,dx = \int \frac{e^x}{(e^x)^2+1} \overset{\uparrow}{=} \int \frac{1}{t^2+1}\,dt =$

$= \arctan(t)+C = \arctan(e^x)+C$

$t=e^x,\ dt=e^x\,dx$

•• $\int \frac{2x}{(x^2+1)^2}\,dx \overset{\uparrow}{=} \int \frac{1}{(t+1)^2} = \int (t+1)^{-2}\,dt =$

$= \frac{(t+1)^{-1}}{-1}+C = -\frac{1}{x^2+1}+C$

$t=x^2,\ dt=2x\,dx$

בס״ד

$$\int \sqrt{x^3+4}\cdot x^5\,dx = \int \sqrt{t}\,(t-4)\cdot\frac{1}{3}\,dt =$$

$$t=x^3+4\ ,\quad x^3=t-4\ ,\quad dt=3x^2dx$$

$$=\frac{1}{3}\left(\int\sqrt{t}\cdot t-\int 4\sqrt{t}\right)=$$

$$=\frac{1}{3}\left(\frac{2t^{\frac{5}{2}}}{5}-4\cdot\frac{2}{3}\cdot t^{\frac{3}{2}}\right)=\frac{2}{3}\left(\frac{(x^3+4)^{\frac{5}{2}}}{5}-\frac{4}{3}(x^3+4)^{\frac{3}{2}}\right)+C$$

$$\int e^{\sqrt[3]{x}}\,dx = 3\int e^t\cdot t^2\,dt =$$

$$t=x^{\frac{1}{3}}\ ,\quad t^2=x^{\frac{2}{3}}\ ,\quad 3t^2dt=dx$$

= עושים אינטגרציה בחלקים פעמיים. ראה לעיל.

$$\int \ln^3 x\,dx = \int t^3\cdot e^t\,dt \leftarrow$$ אינטגרציה בחלקים 3 פעמים

$$t=\ln x\ ,\quad dt=\frac{1}{x}dx\ ,\quad x\cdot dt=dx,\quad x=e^t$$

$$\int\frac{1}{\sqrt{x(4+x)}}\,dx=\int\frac{1}{\sqrt{x}\cdot\sqrt{4+x}}=2\int\frac{1}{\sqrt{4+t^2}}\,dt=$$

$$t=\sqrt{x}\ ,\quad t^2=x\ ,\quad dt=\frac{1}{2\sqrt{x}}dx$$

$$=2\int\frac{1}{2\sqrt{1+\frac{t^2}{4}}}=\int\frac{1}{\sqrt{1+\left(\frac{t}{2}\right)^2}}=$$

$$u=\frac{t}{2}\ ,\quad 2du=dx$$

$$=2\int\frac{1}{\sqrt{1+u^2}}\,du=2\arcsin(u)=2\arcsin\left(\frac{t}{2}\right)=2\arcsin\left(\frac{\sqrt{x}}{2}\right)+C$$

אינטגרציה בחלקים:

- $\boxed{\int x\cdot\cos x\,dx} = x\cdot\sin x-\int\sin x\,dx =$

$$f'=\cos x \quad g=x$$
$$f=\sin x \quad g'=1$$

$= x\cdot\sin x+\cos x+c$

- $\boxed{\int x\cdot\ln x\,dx} = \frac{x^2}{2}\ln x-\int\frac{x^2}{2}\cdot\frac{1}{x} =$

$$f'=x \quad g=\ln x$$
$$f=\frac{x^2}{2} \quad g'=\frac{1}{x}$$

$= \frac{x^2}{2}\ln x-\frac{x^2}{4}+c$

- $\boxed{\int \ln x\,dx} = x\ln x-\int 1dx = x\ln x-x+c$

$$f'=1 \quad g=\ln x$$
$$f=x \quad g=\frac{1}{x}$$

- $\boxed{\int x^4\ln x\,dx} = \frac{x^5}{5}\ln x-\int\frac{x^5}{5}\cdot\frac{1}{x} = \frac{x^5}{5}\ln x-\frac{1}{5}\cdot\frac{x^5}{5}+c$

$$f'=x^4 \quad g=\ln x$$
$$f=\frac{x^5}{5} \quad g'=\frac{1}{x}$$

- $\boxed{\int x\arctan(x)\,dx} = \frac{x^2}{2}\arctan x-\int\frac{x^2}{2}\cdot\frac{1}{x^2+1} =$

$$f'=x \quad g=\arctan(x)$$
$$f=\frac{x^2}{2} \quad g'=\frac{1}{1+x^2}$$

$= \frac{x^2}{2}\arctan x-\frac{1}{2}\int\frac{x^2}{x^2+1} =$

$$\left[\int\frac{x^2}{x^2+1}=\int\frac{x^2+1-1}{x^2+1}=\int 1-\frac{1}{x^2+1}=x-\arctan x\right]$$

$= \frac{x^2}{2}\arctan x-\frac{1}{2}(x-\arctan x)+c$

בס״ד

$$\int e^x \cos x \, dx \overset{\circ}{=} e^x \cdot \sin(x) - \int e^x \cdot \sin x \overset{\circ}{=}$$

$$f' = \cos x \quad g = e^x \qquad f = \sin x \quad g' = e^x$$

$$f' = \sin x \quad g = e^x \qquad f = -\cos x \quad g' = e^x$$

$$= e^x \cdot \sin x - \left(-e^x \cos x - \int e^x (-\cos x)\right) =$$

$$= e^x \cdot \sin x + e^x \cos x - \int e^x \cos x \Rightarrow$$

הגענו לזה הראשון.
נעביר אגף ונחלק ב 2

$$\Rightarrow \int e^x \cos x = \frac{1}{2} e^x (\sin x + \cos x) + C$$

$$\int e^{\sqrt{x}} \, dx \overset{\circ}{=} \int e^t \, 2\sqrt{x} \, dt = \int e^t \cdot 2t \, dt \overset{\circ}{=}$$

$$t = \sqrt{x}, \quad dt = \frac{dx}{2\sqrt{x}}, \quad 2\sqrt{x} \, dt = dx$$

$$f' = e^t \quad g = t \qquad f = e^t \quad g' = 1$$

$$= 2\left(e^t \cdot t - \int e^t \cdot 1\right) = 2te^t - 2e^t = 2\sqrt{x} e^{\sqrt{x}} - 2e^{\sqrt{x}} + C$$

$$\int x^2 e^{-4x} \, dx \overset{\circ}{=} -\frac{x^2 \cdot e^{-4x}}{4} + \frac{2}{4} \int e^{-4x} \cdot x \overset{\circ}{=}$$

$$f' = e^{-4x} \quad g = x^2 \qquad f = \frac{e^{-4x}}{-4} \quad g' = 2x$$

$$f' = e^{-4x} \quad g = x \qquad f = \frac{e^{-4x}}{-4} \quad g' = 1$$

$$= \frac{-x^2 \cdot e^{-4x}}{4} + \frac{1}{2}\left(\frac{e^{-4x} \cdot x}{-4} + \frac{1}{4} \int e^{-4x}\right) \qquad \frac{e^{-4x}}{4} + C$$

$$\int \frac{x}{\cos^2 x} \, dx \overset{\circ}{=} x \cdot \tan x - \int \tan x =$$

ראה לעיל

$$f' = \frac{1}{\cos^2 x} \quad g = x \qquad f = \tan x \quad g' = 1$$

$$= x \tan x + \ln|\cos x| + C$$

בסוף

$$\bullet\bullet \int x^2 \sin(2x)\, dx$$

$$= \frac{-x^2\cdot\cos(2x)}{2} + \int \cos(2x)\cdot x =$$

$f' = \sin(2x)$ $\quad g = x^2$

$f = \frac{-\cos(2x)}{2}$ $\quad g' = 2x$

$f' = \cos(2x)$ $\quad g = x$

$f = \frac{\sin(2x)}{2}$ $\quad g' = 1$

$$= \frac{-x^2\cos(2x)}{2} + \frac{x\cdot\sin(2x)}{2} - \frac{1}{2}\int \sin(2x) =$$

$$= \frac{1}{2}\left(-x^2\cos(2x) + x\cdot\sin(2x) + \frac{1}{2}\cos(2x)\right) + C$$

$$\bullet\bullet \int \frac{xe^x}{(x+1)^2}\, dx$$

$$= \frac{-xe^x}{x+1} + \int \frac{e^x(x+1)}{x+1} =$$

$f' = \frac{1}{(x+1)^2}$ $\quad g = xe^x$

$f = -\frac{1}{x+1}$ $\quad g' = e^x + xe^x$

$$= \frac{-xe^x}{x+1} + \int e^x =$$

$$= \frac{-xe^x}{x+1} + e^x + C = \frac{e^x}{x+1} + C$$

$$\bullet\bullet \int \ln^2 x\, dx$$

$$= \ln x\cdot(x\ln x - x) - \int \frac{x\ln x - x}{x} =$$

$f' = \ln x$ $\quad g = \ln x$

ראה לעיל $\quad f = x\ln x - x$ $\quad g' = \frac{1}{x}$

$$= \ln x(x\ln x - x) - \int(\ln x - 1) = x\ln^2 x - x\ln x - x\ln x + x + x =$$

$$= x\ln^2 x - 2x\ln x + 2x + C$$

בס״ד

$$\cdots \boxed{\int x\tan^2 x\,dx} = \int x\cdot\frac{1-\cos^2 x}{\cos^2 x} =$$

כמו $\int tg$

$$= \int x\left(\frac{1}{\cos^2 x}-1\right) = \int \frac{x}{\cos^2 x} - x = x\tan x + \ln|\cos x| - \frac{x^2}{2} + C$$

$$\cdots \boxed{\int \arcsin\left(\sqrt{\frac{x}{x+1}}\right)dx} = \overset{*}{x\cdot\arcsin\sqrt{\frac{x}{x+1}}} - \int \frac{x\cdot\sqrt{\frac{x}{(x+1)^2}}}{2\cdot x} =$$

$f'=1 \quad g=\arcsin\sqrt{\frac{x}{x+1}}$

$f=x \quad g'=\frac{\sqrt{\frac{x}{(x+1)^2}}}{2x}$

$t=\sqrt{x}\ ,\ 2\sqrt{x}\,dt=dx\ ,\ x=t^2$

$$= * - \int \frac{\sqrt{x}}{2|x+1|}dx = * - \frac{1}{2}\int \frac{t}{2(t^2+1)}\cdot 2\sqrt{x}\,dt = * - \frac{1}{2}\int \frac{t^2}{t^2+1}dt =$$

$$= * - \frac{1}{2}\int 1 - \frac{1}{t^2+1} = x\cdot\arcsin\sqrt{\frac{x}{x+1}} - \frac{1}{2}\left(\sqrt{x} - \arctan\sqrt{x}\right) + C$$

$$\cdots \boxed{\int (x-2)^2\cdot\sqrt{x+3}\,dx} = \frac{2}{3}(x+3)^{\frac{3}{2}}(x-2)^2 - \frac{4}{3}\int (x+3)^{\frac{3}{2}}(x-2) \Rightarrow$$

$f'=(x+3)^{\frac{1}{2}} \quad g=(x-2)^2$

$f=\frac{2}{3}(x+3)^{\frac{3}{2}} \quad g'=2(x-2)$

$f'=(x+3)^{\frac{3}{2}} \quad g=x-2$

$f=\frac{2}{5}(x+3)^{\frac{5}{2}} \quad g'=1$

$$\Rightarrow \int (x+3)^{\frac{3}{2}}(x-2) = \frac{2}{5}(x+3)^{\frac{5}{2}}(x-2) - \frac{2}{5}\int (x+3)^{\frac{5}{2}}$$

סה״כ:

$$\frac{2}{3}(x+3)^{\frac{3}{2}}(x-2)^2 - \frac{4}{3}\left[\frac{2}{5}\left((x+3)^{\frac{5}{2}}(x-2) - \frac{2}{7}(x+3)^{\frac{7}{2}}\right)\right] + C =$$

$$= \frac{2}{3}(x+3)^{\frac{3}{2}}(x-2)^2 - \frac{8}{15}(x+3)^{\frac{5}{2}}(x-2) + \frac{16}{105}(x+3)^{\frac{7}{2}} + C$$

פונקציה רציונלית:

חילוק פולינומים:

$$\frac{3x^5-4x^3+2x^2+3}{x^3+1} = 3x^2-4+\frac{7-x^2}{x^3+1}$$

$$\frac{2x^4+3x^3-x^2+2}{x^2+2x+2} = 2x^2-x-3+\frac{8(x+1)}{x^2+2x+2}$$

$$\frac{-x^7-3x^6+4x^4-2x^3+6x}{x^5-2x^3+x^2-4} = -x^2-3x-2\;\frac{-x^4-3x^3-2x^2-6x-8}{x^5-2x^3+x^2-4}$$

פירוק לשברים חלקיים:

$$\frac{1}{x^2+x} = \frac{1}{x}-\frac{1}{x+1}$$

$$\frac{x-14}{x^2-2x-8} = \frac{3}{x+4}-\frac{2}{x-2}$$

$$\frac{8x^2-31x+9}{(x+1)(x-3)^2} = \frac{5}{x-3}-\frac{3}{(x-3)^2}+\frac{3}{x+1}$$

$$\frac{5x^3+5x^2+x-1}{x^2(x^2+x+1)} = \frac{2}{x}-\frac{1}{x^2}+\frac{3x+4}{x^2+x+1}$$

$$\frac{3x^4-2x^3+x-9}{(x^2+1)^2\cdot(x-2)} = \frac{2x+2}{x^2+1}+\frac{3}{(x^2+1)^2}+\frac{1}{x-2}$$

• $\int \frac{1}{x^2-4}\,dx = \int \frac{1}{(x-2)(x+2)} = \frac{1}{4}\int\left(\frac{1}{x-2} - \frac{1}{x+2}\right)dx =$ בסכ

$= \frac{1}{4}\left(\int\frac{1}{x-2} - \int\frac{1}{x+2}\right) = \frac{1}{4}\cdot\left(\ln|x-2| - \ln|x+2|\right) + c$

• $\int \frac{2-2x}{x^2-4}\,dx = -2\int\frac{x-1}{(x-2)(x+2)} = -2\int\frac{x-2}{(x-2)(x+2)} + \frac{1}{(x-2)(x+2)} =$

$= -2\left(\int\frac{1}{x+2} + \int\frac{1}{(x-2)(x+2)}\right) = -2\left(\ln|x+2| + \frac{1}{4}\left(\ln|x-2| - \ln|x+2|\right)\right) + c$

• $\int \frac{4x-1}{x^2+1}\,dx = 2\int\frac{2x}{x^2+1} - \int\frac{1}{x^2+1} = 2\ln|x^2+1) - \arctan x + c$ לפי הצבה

•• $\int \frac{4x+2}{x^2+4}\,dx = 2\left(\int\frac{2x}{x^2+4} + \int\frac{1}{x^2+4}\right) =$

$= 2\ln|x^2+4| + 2\int\frac{1}{4\left(\frac{x^2}{4}+1\right)} = 2\ln|x^2+4| + \frac{1}{2}\int\frac{1}{\left(\frac{x}{2}\right)^2+1} =$ הצבה

$= 2\ln|x^2+4| + \arctan\left(\frac{x}{2}\right) + c$

•• $\int \frac{x^4+3x^3+2x^2+16x-6}{x^2+4}\,dx = \int\left(x^2+3x-2 + \frac{4x+2}{x^2+4}\right)dx$

מ״די ראה לעיל

X • $\int \frac{8x}{(x-2)^2(x+2)}\,dx = \int\frac{1}{x-2} + \int\frac{4}{(x-2)^2} - \int\frac{1}{x+2} =$ מ״די

• $\int \frac{2x^2+x-1}{(x^2+1)(x-3)}\,dx = \int\frac{1}{x^2+1} + \int\frac{2}{x-3} =$ מ״די

• $\int \frac{3x^5-10x^3-4x^2-9x+18}{x^2-4}\,dx = \int\left(3x^3+2x-4 - \frac{1}{x+2}\right) =$ מ״די

בס״ד

•• $\int \frac{x-2}{x^2+2x+3}\,dx = \frac{1}{2}\int \frac{2x+2-6}{x^2+2x+3} = \frac{1}{2}\int \frac{2x+2}{x^2+2x+3} - \int \frac{6}{x^2+2x+3}$

$(x+1)^2+2$

$= \frac{1}{2}\ln|x^2+2x+3| - 3\int \frac{1}{2\left(\frac{(x+1)^2}{2}+1\right)} =$ *

$= * - \frac{3}{2}\int \frac{1}{\left(\frac{x+1}{\sqrt{2}}\right)^2+1} \overset{t=\frac{x+1}{\sqrt{2}}}{=} * - \frac{3}{2}\int \sqrt{2}\cdot\frac{1}{t^2+1}\,dt =$

$= \frac{1}{2}\left(\ln|x^2+2x+3| - 3\sqrt{2}\arctan\left(\frac{x+1}{\sqrt{2}}\right)\right)+C$ סה״כ:

•• $\int \frac{1}{(2x+3)^3}\,dx = \int \frac{1}{2^3\left(x+\frac{3}{2}\right)^3} = \frac{1}{2^3}\cdot\int\left(x+\frac{3}{2}\right)^{-3} = \frac{1}{2^3}\cdot\frac{\left(x+\frac{3}{2}\right)^{-2}}{-2}+C$

••• $\int \frac{4x-2}{3x^2+3x+6}\,dx = \frac{2}{3}\int \frac{2x-1}{x^2+x+2} = \frac{2}{3}\left(\int \frac{2x}{x^2+x+2} - \int \frac{1}{x^2+x+2}\right) =$

$= \frac{2}{3}\left(\ln|x^2+x+2| - \int \frac{1}{\left(x+\frac{1}{2}\right)^2+\frac{7}{4}}\right) = * - \frac{4}{7}\int \frac{1}{\frac{4}{7}\left(x+\frac{1}{2}\right)^2+1} =$ *

$= * - \frac{4}{7}\int \frac{1}{\left(\frac{2}{\sqrt{7}}\left(x+\frac{1}{2}\right)\right)^2+1} \overset{t=\frac{2x+1}{\sqrt{7}}}{=} * - \frac{4}{7}\int \frac{\sqrt{7}}{2}\cdot\frac{1}{t^2+1}\,dt = \frac{-2}{\sqrt{7}}\cdot\arctan(t) =$

$= \frac{2}{3}\left(\ln|x^2+x+2| - \frac{2}{\sqrt{7}}\arctan\left(\frac{2x+1}{\sqrt{7}}\right)\right)+C$ סה״כ:

המשך

$$\bullet\bullet\bullet \boxed{\int \frac{2x+5}{(x^2-2x+1)^4}dx} = \int \frac{2x-2}{(x^2-2x+1)^4} + \int \frac{7}{(x^2-2x+1)^4} =$$

$$\underset{t=x^2-2x+1}{=} \int \frac{1}{t^4}dt + 7\int \frac{1}{(x-1)^8} = -\frac{t^{-3}}{3} + 7\cdot\frac{(x-1)^{-7}}{-7} =$$

$$= -\frac{1}{3(x^2-2x+1)^3} - \frac{1}{(x-1)^7} =$$

$$= -\frac{1}{3(x-1)^6} - \frac{1}{(x-1)^7} = -\frac{x+2}{3(x-1)^7} + C$$

$$\bullet\bullet\bullet \boxed{\int \frac{4x^3+4x+3}{x^4+2x^2+3x}dx} = \int \frac{4x^3+4x-2x+3-2}{x^4+2x^2+3x} =$$

$$= \int \frac{4x^3+4x+3}{x^4+2x^2+3x} - \int \frac{2}{x(x^2-x+3)} =$$

$$= \ln|x^4+2x^2+3x| + \int \frac{2(x-1)}{3(x^2-x+3)} - \frac{2}{3x} =$$

$$= \ln|x^4+2x^2+3x| + \frac{2}{3}\int \frac{x-1}{x^2-x+3} - \frac{2}{3}\int \frac{1}{x}$$

$$* \int \frac{x-1}{x^2-x+3} = \frac{1}{2}\int \frac{2x-1-1}{x^2-x+3} = \frac{1}{2}\left(\ln|x^2-x+3| - \int \frac{1}{x^2-x+3}\right)$$

$$** \int \frac{1}{x^2-x+3} = \int \frac{1}{(x-\frac{1}{2})^2+\frac{13}{4}} = \frac{4}{13}\int \frac{1}{\left(\sqrt{\frac{4}{13}}(x-\frac{1}{2})\right)^2+1} = \frac{4}{13}\int \sqrt{\frac{13}{4}}\cdot\frac{1}{t^2+1}dt$$

סה"כ:

$$\ln|x^4+2x^2+3x| - \frac{2}{3}\ln|x| + \frac{1}{3}\left[\ln|x^2-x+3| - \frac{1}{\sqrt{13}}\cdot\arctan\left(\sqrt{\frac{4}{13}}\left(x-\frac{1}{2}\right)\right)\right] + C$$