תרגיל 14 – לוגיקה

הי לכולם!

בתרגיל הנוכחי אנחנו הולכים לתרגל את טכניקת ה- .C.P. נעשה זאת בשני חלקים: חלק ראשון – בתחשיב הפסוקים; חלק שני – בתחשיב הפרדיקטים.

חלק ראשון

הוכיחו באמצעות כלל ה- C.P (ולא בשום דרך אחרת!) את הטיעונים הבאים:

.1

$$1.\ N \to W$$
$$2.\ B \to S \quad / \therefore (N \cdot B) \to (W \cdot S)$$

.2

$$1.\ T \to B$$
$$2.\ M \to D \quad / \therefore (T \vee M) \to (B \vee D)$$

.3

$$1.\ E \to N$$
$$2.\ E \to (N \to M)$$
$$3.\ N \to (M \to B) \quad / \therefore E \to B$$

.4

$$1.\ R \to (T \vee S)$$
$$2.\ N \to (S \vee B)$$
$$3.\ \sim S \quad / \therefore (\sim T \cdot \sim B) \to (\sim R \cdot \sim N)$$

הערות:

א. שימו לב שאת רוב הטיעונים כבר הוכחנו פעם; ועכשיו תחזרו לשם ותראו עד כמה ארוכה הייתה ההוכחה (אלה המקומות בהם נעזרנו בטענות עזר), ותראו כמה פשוט זה נהייה עכשיו... =)

ב. בסוף הקובץ הזה אני מצרפת פתרון לתרגיל הראשון. זה במיוחד עבור אלה שלא היו היום, או אלה שלא ממש זוכרים/מבינים מה צריך לעשות. בכל אופן – תנסו קודם לבד ורק אז תציצו =)

חלק שני

בהמשך מצורפים עמודים מהספר של קופי באנגלית. אני רוצה שתעשו את התרגילים הבאים מעמודים 84 – 85:

2,5,6,12,14,15

כאשר את שלושת האחרונים תעשו באמצעות CP ולא בשום דרך אחרת!

הערות:

א. שימו לב שזאת הצרנה עם כמתים $(x),(\exists x)$ ולא פסוקים!
ב. בתרגיל 5, Brown זה שם פרטי ולכן יוצרן כאות קטנה ולא כ Bx (או משהו כזה...)
ג. בתרגיל 6, למשל, כשאומרים "Any navigator has a pistol", יש להצרין את זה עם כמת כולל ולא כמת יישי!
ד. כמו שאמרנו בכיתה: כשאתם מצרינים, כל מונח מקבל הצרנה לפי האות הראשונה שלו. למשל: את Crows נצרין כ- Cx.

עבודה נעימה!

4. Existential Instantiation. One further quantification rule is required. The existential quantification of a propositional function asserts that there exists at least one individual the substitution of whose name for the variable 'x' in that propositional function will yield a true substitution instance of it. Of course we may not know anything else about that individual. But we can take any individual constant, say 'w', which has had no prior occurrence in that context and use it to denote the individual, or one of the individuals, whose existence has been asserted by the existential quantification. Knowing that there is such an individual, and having agreed to denote it by 'w', we can infer from the existential quantification of a propositional function the substitution instance of that propositional function with respect to the individual symbol 'w'. We add as our final quantification rule the principle that from the existential quantification of a propositional function we may validly infer the truth of its substitution instance with respect to an individual constant which has no prior occurrence in that context. The new argument form may be written as

$$\frac{(\exists x)\Phi x}{\therefore\ \Phi z}$$ (where 'z' is an individual constant having no prior occurrence in the context.)

It will be referred to as the 'principle of Existential Instantiation' and abbreviated as '**EI**'.

We make use of the last two quantification rules in constructing a formal proof of validity for the argument: 'All dogs are carnivorous; some animals are dogs; therefore some animals are carnivorous'.

1.	$(x)[Dx \supset Cx]$	
2.	$(\exists x)[Ax \cdot Dx]$	$/\therefore (\exists x)[Ax \cdot Cx]$
3.	$Aw \cdot Dw$	2, **EI**
4.	$Dw \supset Cw$	1, **UI**
5.	$Dw \cdot Aw$	3, Com.
6.	Dw	5, Simp.
7.	Cw	4, 6, M.P.
8.	Aw	3, Simp
9.	$Aw \cdot Cw$	8, 7, Conj.
10.	$(\exists x)[Ax \cdot Cx]$	9, **EG**

We can show the need for the indicated restriction on the use of **EI** by considering the obviously invalid argument: 'Some cats are animals; some dogs are animals; therefore some cats are dogs'. If we ignored the restriction on **EI** that the substitution instance inferred by it can contain only an individual constant which had no prior occurrence in the context, we might be led to construct the following 'proof':

1.	$(\exists x)[Cx \cdot Ax]$	
2.	$(\exists x)[Dx \cdot Ax]$	$/\therefore (\exists x)[Cx \cdot Dx]$
3.	$Cw \cdot Aw$	1, **EI**
4.	$Dw \cdot Aw$	2, **EI** (erroneous)
5.	Cw	3, Simp.
6.	Dw	4, Simp.
7.	$Cw \cdot Dw$	5, 6, Conj.
8.	$(\exists x)[Cx \cdot Dx]$	7, **EG**

The mistake here occurs at step 4. The second premiss assures us that there is at least one thing which is both a dog and an animal. But we are not free to use the symbol 'w' to denote that thing because 'w' has already been used to denote one of the things asserted by the first premiss to be both a cat and an animal. To avoid errors of this sort we must obey the indicated restriction in using **EI**. It should be clear that whenever we use both **EI** and **UI** in a proof to instantiate with respect to the same individual constant, we must use **EI** first.

5. Strengthened Rule of Conditional Proof. In the preceding chapter the method of Conditional Proof was applied only to arguments whose conclusions were conditional in form. But now we must consider arguments whose conclusions are non-compound general propositions. To deal with them we shall strengthen our rule of Conditional Proof and thereby give it wider applicability.

To formulate our strengthened rule of Conditional Proof we must adopt a new method of writing out proofs which make use of the Conditional Method. As explained in the preceding chapter, we used the method of Conditional Proof to establish the validity of an argument having a conditional as conclusion by adding the antecedent of that conditional to the argument's premisses as an assumption, and then deducing the conditional's consequent. The old notation involved the use of an additional slant line and an extra *therefore* sign, as in proving the validity of the argument '$A \supset B \therefore A \supset (A \cdot B)$' by the following four step proof (as explained in Section II of Chapter Two):

1. $A \supset B / \therefore A \supset (A \cdot B)$	
2. A $/ \therefore A \cdot B$	
3. B	1, 2, M.P.
4. $A \cdot B$	2, 3, Conj.

A Conditional Proof of validity for that same argument is set down in our new notation as the following sequence of five steps:

1. $A \supset B / \therefore A \supset (A \cdot B)$	
2. A	assumption
3. B	1, 2, M.P.
4. $A \cdot B$	2, 3, Conj.
5. $A \supset (A \cdot B)$	2–4, C.P.

Here the fifth step is inferred not from any one or two of the preceding steps but from the *sequence* of steps 2, 3, 4, which constitutes a valid deduction of step 4 from steps 1 and 2. In step 5 we infer the validity of the argument '$A \supset B \therefore A \supset (A \cdot B)$' from the demonstrated validity of the argument '$A \supset B, A \therefore A \cdot B$'. That inference is 'justified' by noting the sequence of steps to which appeal is made, and using the letters 'C.P.' to show that the principle of Conditional Proof is being used.

In the second of the preceding proofs, step 2, the assumption, has steps 3 and 4 dependent upon it. Step 5, however, is *not* dependent upon step 2, but only upon step 1. Step 5 is therefore *outside* or *beyond the scope* of the assumption made as step 2. When an assumption is made in a Conditional Proof of validity, its 'scope' is always *limited*, never extending all the way to the last line of the demonstration.

A notation is now introduced which is very helpful in keeping track of assumptions and their *scopes*. A bent arrow is used for

this purpose, with its head pointing at the assumption from the left, its shaft bending down to run along all steps within the scope of the assumption, and then bending inward to mark the end of the scope of that assumption. The scope of the assumption in the preceding proof is indicated thus:

1. $A \supset B / \therefore A \supset (A \cdot B)$	
→2. A	assumption
3. B	1, 2, M.P.
4. $A \cdot B$	2, 3, Conj.
5. $A \supset (A \cdot B)$	2–4, C.P.

It should be observed that only a line inferred by the principle of Conditional Proof ends the scope of an assumption, and that every use of the rule of Conditional Proof serves to end the scope of an assumption. If the scope of an assumption does *not* extend all the way to the end of a proof, then the final step of the proof does not *depend* on that assumption, but has been proved to follow from the original premisses alone. Hence we need not restrict ourselves to using as assumptions only the antecedents of conditional conclusions. *Any* proposition can be taken as an assumption of limited scope, for the final step which is the conclusion will always be beyond its scope and independent of it.

A more complex demonstration which involves making *two* assumptions is the following (incidentally, when our bent arrow notation is used, the word 'assumption' need not be written, since each assumption is sufficiently identified as such by the arrowhead on its left):

1. $(A \vee B) \supset [(C \vee D) \supset E] / \therefore A \supset (C \supset E)$	
→2. A	
3. $A \vee B$	2, Add.
4. $(C \vee D) \supset E$	1, 3, M.P.
→5. C	
6. $C \vee D$	5, Add.
7. E	4, 6, M.P.
8. $C \supset E$	5–7, C.P.
9. $A \supset (C \supset E)$	2–8, C.P.

In this proof, lines 2 through 8 lie within the scope of the first assumption, while lines 5, 6, and 7 lie within the scope of the second assumption. From these examples it is clear that the scope of an assumption in a proof contains all lines from itself down *to* the line which is inferred by C.P. from the sequence of steps beginning with that assumption. In the preceding proof, the second assumption lies within the scope of the first because it lies between the first assumption and step 9 which is inferred by C.P. from the sequence of steps 2 through 8.

When we use this new method of writing out a Conditional Proof of validity the scope of every original premiss extends all the way to the end of the proof. The original premisses may be supplemented by additional assumptions provided that the latter's scopes are limited and do not extend to the end of the proof. Each step of a formal proof of validity must be either a premiss, or an assumption of limited scope, or must follow validly from one or two preceding steps by an elementary valid argument form, or must follow from a *sequence* of preceding steps by the principle of Conditional Proof.

It should be remarked that the strengthened principle of Conditional Proof includes the method of Indirect Proof as a special case. Since any assumption of limited scope may be made in a Conditional Proof of validity, we can take as our assumption the negation of the argument's conclusion. Once a contradiction is obtained, we can *continue on through* the contradiction to obtain the desired conclusion by Addition and the Disjunctive Syllogism.* Once that is done, we can use the rule of Conditional Proof to end the scope of that assumption and obtain a conditional whose consequent is the argument's conclusion and whose antecedent is the negation of that conclusion. And from such a conditional the argument's conclusion will follow by Implication, Double Negation, and Tautology.

Any assumption of limited scope may be made in a Conditional Proof of validity, and in particular we are free to make an assumption of the form 'Φy'. Thus the argument 'All freshmen and sophomores are invited and will be welcome; therefore all freshmen are invited' may be proved valid by the following Conditional Proof:

* As explained on page 56.

1. $(x)[(Fx \vee Sx) \supset (Ix \cdot Wx)] / \therefore (x)[Fx \supset Ix]$	
→2. Fy	
3. $(Fy \vee Sy) \supset (Iy \cdot Wy)$	1, UI
4. $Fy \vee Sy$	2, Add.
5. $Iy \cdot Wy$	3, 4, M.P.
6. Iy	5, Simp.
7. $Fy \supset Iy$	2–6. C.P.
8. $(x)[Fx \supset Ix]$	7, UG

More than one assumption of limited scope can be made in proving the validity of arguments involving quantifiers, as in the following Conditional Proof:

1. $(x)[(Ax \vee Bx) \supset (Cx \cdot Dx)]$	
2. $(x)\{(Cx \vee Ex) \supset [(Fx \vee Gx) \supset Hx\} / \therefore (x)\{Ax \supset (Fx \supset Hx)\}$	
3. $(Ay \vee By) \supset (Cy \cdot Dy)$	1, UI
4. $(Cy \vee Ey) \supset [(Fy \vee Gy) \supset Hy]$	2, UI
→5. Ay	
6. $Ay \vee By$	5, Add.
7. $Cy \cdot Dy$	3, 6, M.P.
8. Cy	7, Simp.
9. $Cy \vee Ey$	8, Add.
10. $(Fy \vee Gy) \supset Hy$	4, 9, M.P.
→11. Fy	
12. $Fy \vee Gy$	11, Add.
13. Hy	10, 12, M.P.
14. $Fy \supset Hy$	11–13, C.P.
15. $Ay \supset (Fy \supset Hy)$	5-14, C.P.
16. $(x)[Ax \supset (Fx \supset Hx)]$	15, UG

EXERCISES

Construct formal proofs of validity for the following arguments, using the rule of Conditional Proof wherever its application will result in a shorter proof:

1. No anarchists are bankers. Jones is an anarchist. Therefore Jones is not a banker.
2. All contestants were deceived. Some contestants were engineers. Therefore some engineers were deceived.
3. No fool is a gentleman. Some hunters are gentlemen. Therefore some hunters are not fools.
4. All insurrectionists were jailed. Some insurrectionists were not killed. Therefore some who were jailed were not killed.
5. All leaders are masterful. Brown is not masterful. Therefore Brown is not a leader.
6. Only officers are navigators. Officers all have pistols. Therefore any navigator has a pistol.
7. Reformers are never quiet. None but reformers have been saints. Therefore there have been no quiet saints.
8. There are useful tautologies. Whatever has use has value. Therefore not all tautologies are valueless.
9. All adolescents are bumptious. No cosmopolitan is bumptious. Some debutantes are cosmopolitans. Therefore some debutantes are not adolescents.
10. The English are friendly. Only the generous are friendly. To be generous one must be honest. There are dishonest industrialists. Therefore not all industrialists are English.
11. All airplanes are fast and convenient. Some airplanes are delapidated. Therefore some delapidated things are fast.
12. Snakes and lizards are reptiles. Reptiles and birds are oviparous. Therefore snakes are oviparous.
13. Every citizen is either a patriot or a traitor. Every patriot is honorable. Some citizens are not honorable. Therefore some citizens are traitors.
14. All members who were present were both surprised and resentful. All members who were interested were present. Therefore all members who were interested were surprised.
15. All crows are black. All black crows are pests. Therefore all crows are black pests.
16. All people are consumers. Some people are wealthy. All wealthy consumers are extravagant. Therefore some people are extravagant.
17. Waitresses and helpers will be discharged if they are clumsy or inefficient. Therefore any waitress will be discharged if she is clumsy.

18. All terriers are lively. Lively terriers are all courageous. Any lively terrier which is courageous is a hunter. Therefore all terriers are hunters.
19. Painters and sculptors are artists. All artists and radicals are Bohemians. Bohemians are unconventional and original. Some painters are neither original nor talented. Therefore some sculptors are talented but not unconventional.
20. No man who is a defendant will be convicted if he is innocent. Any man who is tried is a defendant. Any defendant who is not convicted will be acquitted. Every man who is acquitted is innocent. Therefore any man who is tried will be acquitted if and only if he is innocent.

III. PROVING INVALIDITY

In the preceding chapter we proved the invalidity of invalid arguments containing truth-functional compound statements by assigning truth values to their simple constituent statements in such a way as to make their premisses true and their conclusions false. We can use a very similar method to prove the invalidity of invalid arguments involving quantifiers. The method of proving invalidity about to be described is closely connected with our basic assumption that the universe is non-empty, that is, that there exists at least one individual in the universe.

The assumption that the universe is non-empty could be satisfied in infinitely many different ways: if there is exactly one individual, or if there are exactly two individuals, or if there are exactly three individuals, or etc. For any such case there is a strict logical equivalence between non-compound general propositions and truth-functional compounds of singular propositions. If there is exactly one individual in the universe, say a, then

$$[(x)\Phi x] \equiv \Phi a \quad \text{and} \quad [(\exists x)\Phi x] \equiv \Phi a.$$

If there are exactly two individual in the universe, say a and b, then

$$[(x)\Phi x] \equiv [\Phi a \cdot \Phi b] \quad \text{and} \quad [(\exists x)\Phi x] \equiv [\Phi a \vee \Phi b].$$

פתרון התרגיל הראשון משיעורי הבית:

$1.\ N \rightarrow W$

$2.\ B \rightarrow S \quad / \therefore (N \cdot B) \rightarrow (W \cdot S)$

$3.\ N \cdot B$

$4.\ N \quad 3, Simp$

$5.\ B \quad 3, Simp$

$6.\ W \quad 1,4, M.P.$

$7.\ S \quad 2,5, M.P.$

$8.\ W \cdot S \quad 6,7, Conj$

$9.\ (N \cdot B) \rightarrow (W \cdot S) \quad 3-8, C.P.$