

# פתרון תרגיל בית 13 – לוגיקה

הוכיחו את הטענות הבאים בשיטת הדדוקציה הטבעית:

**א.**

1.  $p \rightarrow (\neg q \rightarrow r)$
2.  $p \wedge \neg r \quad / \therefore q$
3.  $p \quad 2, \text{Simp.}$
4.  $\neg q \rightarrow r \quad 1, 3, \text{M.P.}$
5.  $\neg r \wedge p \quad 2, \text{Com.}$
6.  $\neg r \quad 5, \text{Simp.}$
7.  $\neg \neg q \quad 4, 6, \text{M.T.}$
8.  $q \quad 7, \text{D.N.}$

**ב.**

1.  $p \rightarrow q$
2.  $r \rightarrow s \quad / \therefore (p \rightarrow s) \vee (r \rightarrow q)$
3.  $(p \rightarrow q) \vee (r \rightarrow s) \quad 2, \text{Add.}$
4.  $(\neg p \vee q) \vee (\neg r \vee s) \quad 3, \text{Impl.}$
5.  $(\neg p \vee s) \vee (\neg r \vee q) \quad 4, \text{Assoc. + Com.}$
6.  $(p \rightarrow s) \vee (r \rightarrow q) \quad 5, \text{Impl.}$

**ג.**

1.  $p \vee q$
2.  $p \rightarrow (p \wedge q)$
3.  $\neg p \vee \neg q \quad / \therefore q$
4.  $\neg(p \wedge q) \quad 3, \text{DeM.}$
5.  $\neg p \quad 2, 4, \text{M.T.}$
6.  $q \quad 1, 5, \text{D.S.}$

**.T**

1.  $(p \rightarrow q) \wedge (\neg r \rightarrow \neg s)$
2.  $(\neg s \vee \neg q) \wedge t$
3.  $p \rightarrow \neg t$                        $\therefore \neg p \vee \neg r$
4.  $t \wedge (\neg s \vee \neg q)$     2, *Com.*
5.  $t$                                       4, *Simp.*
6.  $\neg \neg t$                                 5, *D.N.*
7.  $\neg p$                                  3, 6, *M.T.*
8.  $\neg p \vee \neg r$                         7, *Add.*

**.h**

1.  $(p \vee q) \rightarrow r$      $\therefore p \rightarrow r$
2.  $\neg(p \vee q) \vee r$                       1, *Impl.*
3.  $(\neg p \wedge \neg q) \vee r$                     2, *DeM.*
4.  $(\neg p \vee r) \wedge (\neg q \vee r)$     3, *Dist.*
5.  $\neg p \vee r$                                 4, *Simp.*
6.  $p \rightarrow r$                                 5, *Impl.*

**.i**

1.  $s \rightarrow (t \wedge p)$
2.  $(t \vee p) \rightarrow q$      $\therefore s \rightarrow q$
3.  $\neg s \vee (t \wedge p)$                         1, *Impl.*
4.  $(\neg s \vee t) \wedge (\neg s \vee p)$     3, *Dist.*
5.  $\neg s \vee t$                                 4, *Simp.*
6.  $(\neg s \vee t) \vee p$                         5, *Add.*
7.  $\neg s \vee (t \vee p)$                         6, *Assoc.*
8.  $s \rightarrow (t \vee p)$                         7, *Impl.*
9.  $s \rightarrow q$                                 2, 8, *H.S.*